13th CI^{2}MA Focus Seminar
Numerical Methods for Hyperbolic and Related Problems
Universidad de Concepción, July 10, 2017
Auditorio Alamiro Robledo, Facultad de Ciencias Físicas y Matemáticas
Organizers[1]: Raimund Bürger (UdeC) & Luis Miguel Villada (UBB)

Programme

14.30 **Pep Mulet** (Universitat de València, Spain):
Derivatives-free approximate Taylor methods for ODEs
and their relationship with Runge-Kutta methods

15.00 **Enrique D. Fernández-Nieto** (Universidad de Sevilla, Spain):
Finite volume methods for two-layer and two-phase shallow water systems

15.30 **Luis Miguel Villada** (Universidad del Bío-Bío, Concepción):
High-order numerical schemes for one-dimensional non-local conservation laws

16.00 **Natalia Inzunza** (Universidad del Bío-Bío, Concepción):
Convergence of an implicit-explicit scheme
for a two-dimensional parabolic-hyperbolic system

16.30 **Coffee break**

17.00 **Aníbal Coronel** (Universidad del Bío-Bío, Chillán):
Convergence of a second-order level-set algorithm for scalar conservation laws

17.30 **Julio Careaga** (Universidad de Concepción):
Inverse problem of a scalar conservation law modelling sedimentation
in vessels with varying cross-sectional area

18.00 **Mauricio Sepúlveda** (Universidad de Concepción):
On exponential stability for thermoelastic plates—a comparison of different models

18.30 **Raimund Bürger** (Universidad de Concepción):
Non-conforming/DG coupled schemes for multicomponent viscous flow
in porous media with adsorption

20.30 **Seminar Dinner**

[1]This event is supported by Conicyt projects PFB03 (CMM-Basal), PAI/MEC/80150006, and Fondecyt 11140708 and 1170473.

13th CI^{2}MA Focus Seminar
Numerical Methods for Hyperbolic and Related Problems
Universidad de Concepción, July 10, 2017
Supported by Conicyt projects PFB03 (CMM-Basal), PAI/MEC/80150006, Fondecyt 11140708 and 1170473

DERIVATIVES-FREE APPROXIMATE TAYLOR METHODS FOR ODES AND THEIR RELATIONSHIP WITH RUNGE-KUTTA METHODS

PEP MULET

Abstract. We propose a numerical method for ODEs which is based on an approximate formulation of the Taylor methods with a much easier implementation than the original Taylor methods, for the former only require the functions in the ODEs while high order derivatives are require for the latter not their derivatives. In this regard, when Compared to Runge-Kutta methods, the number of function evaluations to achieve a relatively low order is higher, however with the present procedure it is much easier to produce arbitrarily high order schemes. This may be important in some applications where long time precise simulations are crucial. We show also some results related to the stability of the new methods and their link to Runge-Kutta methods.

In many cases the new approach leads to an asymptotically lower computational cost when compared to the Taylor expansion based on exact derivatives. The numerical results that are obtained with our proposal are satisfactory and show that this approximate approach can attain results as good as the exact Taylor procedure with less implementation and computational effort.

This presentation is based on recent joint work with Antonio Baeza and David Zorío, from the University of Valencia and Sebastiano Boscarino and Giovanni Russo, from the University of Catania.

13th CI^{2}MA Focus Seminar
Numerical Methods for Hyperbolic and Related Problems
Universidad de Concepción, July 10, 2017
Supported by Conicyt projects PFB03 (CMM-Basal), PAI/MEC/80150006, Fondecyt 11140708 and 1170473

FINITE VOLUME METHODS FOR TWO-LAYER AND TWO-PHASE SHALLOW WATER SYSTEMS

ENRIQUE D. FERNÁNDEZ-NIETO

Abstract. Several geophysical applications, such as submarine avalanches (see [**?**]), debris flows and sediment transport can be studied by two-layer and two-phase shallow water systems (see [**?**]).

There are several difficulties related to the discretization of these systems, which can be written under the structure of a hyperbolic system with a conservative term, a non-conservative product and source terms. One of them is that the coupling term between the layers or the phases is usually written as a non-conservative product (see [**?**]). All the models considered in this talk include a source term corresponding to a Coulomb friction law. It is multi-evaluated for the case of a material at rest. Finally, some of these models can have complex eigenvalues in some situations.

A finite volume method is considered (see [**?**]), with a special treatment of the Coulomb friction term and the loss of hyperbolicity. Finally, several numerical tests will be presented.

References

[1] F. Bouchut, E.D. Fernández-Nieto, A. Mangeney, and G. Narbona-Reina. A two-phase two-layer model for fluidized granular flows with dilatancy effects. *J. Fluid Mech.*, 801, 166–221, 2016.

[2] E.D. Fernández-Nieto, F. Bouchut, D. Bresch, M.J. Castro, A. Mangeney. A new savage–hutter type model for submarine avalanches and generated tsunami. *J. Comput. Phys.*, 227: 7720–7754, 2008.

[3] E.D. Fernández-Nieto, M.J. Castro, C. Parés. On an intermediate field capturing riemann solver based on a parabolic viscosity matrix for the two-layer shallow water system. *J. Scient. Comp.*, 48: 117–140, 2011.

[4] C. Parés. Numerical methods for nonconservative hyperbolic systems: a theoretical framework. *SIAM J. Num. Analysis* 44: 300–321, 2006.

Departamento de Matemática Aplicada I, Universidad de Sevilla
E-mail address: edofer@gmail.com

13th CI^{2}MA Focus Seminar
Numerical Methods for Hyperbolic and Related Problems
Universidad de Concepción, July 10, 2017
Supported by Conicyt projects PFB03 (CMM-Basal), PAI/MEC/80150006, Fondecyt 11140708 and 1170473

HIGH ORDER NUMERICAL SCHEMES FOR ONE-DIMENSIONAL NON-LOCAL CONSERVATION LAWS

LUIS-MIGUEL VILLADA

Abstract. This talk focuses on the numerical approximation of the solutions of non-local conservation laws in one space dimension. These equations are motivated by two distinct applications, namely a traffic flow model in which the mean velocity depends on a weighted mean of the downstream traffic density [**?**], and a sedimentation model where either the solid phase velocity or the solid-fluid relative velocity depends on the concentration in a neighborhood [**?**]. In both models, the velocity is a function of a convolution product between the unknown and a kernel function with compact support. It turns out that the solutions of such equations may exhibit oscillations that are very difficult to approximate using classical first-order numerical schemes. In [**?**] we considered Discontinuous Galerkin (DG) schemes and Finite Volume WENO (FV-WENO) schemes to obtain high-order approximations. DG schemes give the best numerical results but their CFL condition is very restrictive. On the contrary, FV-WENO schemes can be used with larger time steps. The evaluation of the convolution terms necessitates the use of quadratic polynomials reconstructions in each cell in order to obtain the high-order accuracy with the FV-WENO approach. Simulations using DG and FV-WENO schemes are presented for both applications.

Joint work with: Christophe Chalons (Université Versailles Saint-Quentin-en-Yvelines, France) and Paola Goatin (INRIA Sophia Antipolis - Méditerranée, France.).

References

[1] C. Chalons, P. Goatin, L.M. Villada. *High order numerical schemes for one-dimension non-local conservation laws.* Accepted for publication SIAM Journal on Scientific Computing .

[2] F. Betancourt, R. Bürger, K. Karlsen, E. Tory. *On nonlocal conservation laws modelling sedimentation.* Nonlinearity. **24** (2011), pp 855–885.

[3] B. Cockbur and C-W. Shu. *Runge-Kutta Discontinuous Galerkin methods for convection-dominate problems.* J. Sci. Comput. **16** (2001), pp 173–261.

[4] P. Goatin and S. Scialanga. *Well-posedness and finite volume approximations of the LWR traffic flow model with non-local velocity.* Netw. Heterog. Media, **11**(1) (2016), pp 107–121.

[5] C.-W. Shu.. Essentially non-oscillatory and weighted essentially non-oscillatory schemes for hyperbolic conservation laws . Springer Berlin Heidelberg. (1998), pp 325–432.

Departamento de Matemática-Universidad del Bío-Bío
E-mail address: lvillada@ubiobio.cl

13th CI^{2}MA Focus Seminar
Numerical Methods for Hyperbolic and Related Problems
Universidad de Concepción, July 10, 2017
Supported by Conicyt projects PFB03 (CMM-Basal), PAI/MEC/80150006, Fondecyt 11140708 and 1170473

CONVERGENCE OF AN IMPLICIT-EXPLICIT SCHEME FOR A TWO-DIMENSIONAL PARABOLIC-HYPERBOLIC SYSTEM

NATALIA INZUNZA

Abstract. This talk focuses on the convergence of an Implicit-Explicit Finite Volume scheme arising from discretization in the parabolic-hyperbolic coupled system describing the competition of predator and prey populations in two dimensions. The system proposed in [1], consists of a conservation law with a non-local and non-linear flow for predators, together with a parabolic equation for the prey. The numeric scheme consists of an explicit discretization for the hyperbolic part together with an implicit discretization for the parabolic term. The resulting scheme is a variant of the fully explicit scheme presented in [2]. The convergence of the hyperbolic variable is demonstrated, whereas for the parabolic part only weak* convergence in $\mathbf{L}^\infty$. Simulations are presented describing the characteristic behavior of the predator-prey system, the efficiency and the convergence of the numerical scheme.

Joint work with: Luis-Miguel Villada (Universidad del Bío-Bío).

References

[1] Colombo, R. M. and Rossi, E. *Hyperbolic predators vs. parabolic prey.* Commun. Math. Sci., 13(2) (2015), 369-400.

[2] Rossi, E. Schleper, V. *Convergence of a numerical scheme for a mixed hyperbolic-parabolic system in two space dimensions.* ESAIM Math. Modelling Numer. Anal., vol. 50, pp. 475-497.

[3] R. J. LeVeque. Finite volume methods for hyperbolic problems. Cambridge Texts in Applied Mathematics. Cambridge University Press, Cambridge, 2002.

[4] R. Eymard, T. Gallouet and R. Herbin, Finite Volume Methods, In *Handbook of Numerical Anlysis* (Vol. VII), editors: P.G. Ciarlet and J.L. Lions, pp. 729-1020, North-Holland, pp. 729-1020, 2000.

Departamento de Matemática-Universidad del Bío-Bío
E-mail address: nataliainzunza90@gmail.com

13th CI^{2}MA Focus Seminar
Numerical Methods for Hyperbolic and Related Problems
Universidad de Concepción, July 10, 2017
Supported by Conicyt projects PFB03 (CMM-Basal), PAI/MEC/80150006, Fondecyt 11140708 and 1170473

CONVERGENCE OF A SECOND ORDER LEVEL-SET ALGORITHM FOR SCALAR CONSERVATION LAWS

ANÍBAL CORONEL

Abstract. In this paper we study the convergence of the level-set algorithm introduced by Aslam for tracking the discontinuities in scalar conservation laws in the case of linear or strictly convex flux function [1]. The numerical method is deduced by the level-set representation of the entropy solution: the zero of a level-set function is used as an indicator of the discontinuity curves and two auxiliary states, which are assumed continuous through the discontinuities, are introduced. Following the ideas of [5], we rewrite the numerical level-set algorithm as a procedure consisting of three big steps: (a) initialization, (b) evolution and (c) reconstruction. In (a) we choose an entropy admissible level-set representation of the initial condition. In (b), for each iteration step, we solve an uncoupled system of three equations and select the entropy admissible level-set representation of the solution profile at the end of the time iteration. In (c) we reconstruct the entropy solution by using the level-set representation. Assuming that in the step (b) we can use a second order scheme to approximate each equation of that we prove the convergence of the numerical solution of the level set algorithm to the entropy solution in L^1, using the ideas of Popov and collaborators [2, 3, 4]. In addition, some numerical examples focused on the elementary wave interaction are presented.

This contribution is a joint work with M. Sepúlveda (Concepción, Chile).

References

[1] T. D. Aslam, A level set algorithm for tracking discontinuities in hyperbolic conservation laws I: scalar equations, *J. Comput. Phys.* 167(2)(2001), 413–438.

[2] S. Konyagin, B. Popov, and O. Trifonov, Ognian . On convergence of minmod-type schemes. SIAM J. Numer. Anal. 42 (2005), no. 5, 1978–1997.

[3] B. Popov and O. Trifonov. One-sided stability and convergence of the Nessyahu-Tadmor scheme. Numer. Math. 104 (2006), no. 4, 539–559.

[4] B. Popov and O. Trifonov. Order of convergence of second order schemes based on the minmod limiter. Math. Comp. 75 (2006), no. 256, 1735–1753.

[5] A. Coronel, P. Cumsille, and M. Sepúlveda, Convergence of a level-set algorithm in scalar conservation laws. *Numer. Methods Partial Differential Equations* 31 (2015), no. 4, 1310–1343.

† GMA, Departamento de Ciencias Básicas, Facultad de Ciencias, Universidad del Bío-Bío, Campus Fernando May, Chillán, Chile,
E-mail address: acoronel@ubiobio.cl

13th CI^{2}MA Focus Seminar
Numerical Methods for Hyperbolic and Related Problems
Universidad de Concepción, July 10, 2017
Supported by Conicyt projects PFB03 (CMM-Basal), PAI/MEC/80150006, Fondecyt 11140708 and 1170473

INVERSE PROBLEM OF A SCALAR CONSERVATION LAW MODELLING SEDIMENTATION IN VESSELS WITH VARYING CROSS-SECTIONAL AREA

JULIO CÉSAR CAREAGA

Abstract. The sedimentation of an ideal suspension in a vessel with variable cross-sectional area can be described by an initial-boundary value problem for a scalar nonlinear hyperbolic conservation law with a nonconvex flux function and a weight function that depends on spatial position. The sought unknown is the local solids volume fraction. Solutions exhibit discontinuities that mostly travel at variable speed, i.e., they are curved in the space-time plane as shown in [**?**]. It presents the entropy solution for the conical case and the problem arises from the determination of the flow function that represents the nolineal term within the differential equation using as data one of the discontinuity jumps of the entropy solution. It shown a closed form of resolution of the problem besides the algorithm necessary for the identification from discrete data, experiments with real data and numerical simulations for the identified flux function using the numerical method described in [**?**].

This work has partly been inspired by the inverse problem development in [**?**] and is based on recent joint work [**?**] with Raimund Bürger (Universidad de Concepción, Chile) and Stefan Diehl (Lund University, Sweden).

References

[1] R. Bürger, J. Careaga and S. Diehl. A simulation model for settling tanks with varying cross-sectional area. Preprint 2017-07, Centro de Investigación en Ingeniería Matemática, Universidad de Concepción; submitted.

[2] R. Bürger, J. Careaga and S. Diehl. Entropy solutions of a scalar conservation law modeling sedimentation in vessels with varying cross-sectional area. *SIAM J. Appl. Math.* 77:789-811, 2017.

[3] R. Bürger, J. Careaga and S. Diehl. Flux identification for scalar conservation laws modelling sedimentation in vessels with varying cross-sectional area. Preprint 2016-40, Centro de Investigación en Ingeniería Matemática, Universidad de Concepción; submitted.

[4] R. Bürger and S. Diehl. Convexity-preserving flux identification for scalar conservation laws modelling sedimentation. *Inverse Problems* 29: 045008 (30pp), 2013.

CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción
E-mail address: `juliocareaga@udec.cl`

13th CI^{2}MA Focus Seminar
Numerical Methods for Hyperbolic and Related Problems
Universidad de Concepción, July 10, 2017
Supported by Conicyt projects PFB03 (CMM-Basal), PAI/MEC/80150006, Fondecyt 11140708 and 1170473

ON EXPONENTIAL STABILITY FOR THERMOELASTIC PLATES – A COMPARISON OF DIFFERENT MODELS

MAURICIO SEPÚLVEDA

Abstract. We consider different models of thermoelastic plates in a bounded reference configuration: with Fourier heat conduction or with the Cattaneo model, and with or without inertial terms. Some models exhibit exponential stability, others are not exponential stable. In the cases of exponential stability, we give an explicit estimate for the rate of decay in terms of the essential parameters appearing (delay $\tau \geq 0$, inertial constant $\mu \geq 0$), using multiplier methods. The singular limits $\tau \downarrow 0$, and, in particular, $\mu \downarrow 0$ are also investigated in order to understand the mutual relevance for the (non-) exponential stability of the models. Numerical simulations underline the analytic estimates.

This contribution is based on recent joint work with Jaime E. Muñoz-Rivera (LNCC, Brasil), and Reinhard Racke (University of Konstanz, Germany).

References

[1] M. Alves, J. Muñoz-Rivera, M. Sepúlveda, O. Vera, M. Zegarra, The asymptotic behaviour of the linear transmission problem in viscoelasticity. *Mathematische Nachrichten*, **287**, 5-6 (2014), 483-497.

[2] M. Alves, J. Muñoz-Rivera, M. Sepúlveda, O. Vera, Exponential and the lack of exponential stability in transmission problems with localized Kelvin-Voigt dissipation. *SIAM Journal on Applied Mathematics*, **74**, 2 (2014), 354-365.

[3] Muñoz Rivera, J. E., Racke, R.: Smoothing properties, decay, and global existence of solutions to nonlinear coupled systems of thermoelastic type. *SIAM J. Math. Anal.* **26** (1995), 1547–1563.

[4] Muñoz Rivera, J.E., Racke, R.: Large solutions and smoothing properties for nonlinear thermoelastic systems. *J. Differential Equations* **127** (1996), 454–483.

[5] Quintanilla, R., Racke, R.: Addendum to: Qualitative aspects of solutions in resonators. *Arch. Mech.* **63** (2011), 429–435.

CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción
E-mail address: `msepulveda@ci2ma.udec.cl`

13th CI^{2}MA Focus Seminar
Numerical Methods for Hyperbolic and Related Problems
Universidad de Concepción, July 10, 2017
Supported by Conicyt projects PFB03 (CMM-Basal), PAI/MEC/80150006, Fondecyt 11140708 and 1170473

NON-CONFORMING/DG COUPLED SCHEMES FOR MULTICOMPONENT VISCOUS FLOW IN POROUS MEDIA WITH ADSORPTION

RAIMUND BÜRGER

Abstract. Polymer flooding is an important stage of enhanced oil recovery [5] in petroleum reservoir engineering. A model of this process is based on the study of multicomponent viscous flow in porous media with adsorption. This model can be expressed as a Brinkman-based model of flow in porous media coupled to a system of non-strictly hyperbolic conservation laws having multiple components. The discretisation proposed for this coupled flow-transport problem combines a stabilised non-conforming method for the Brinkman flow problem [4] with a discontinuous Galerkin (DG) method for the transport equations. The DG formulation of the transport problem is based on discontinuous numerical fluxes [2, 6]. An invariant region property is proved under the (mild) assumption that the underlying mesh is a **B**-triangulation [3]. This property states that only physically relevant (bounded and non-negative) saturation and concentration values are generated by the scheme. Numerical tests illustrate the accuracy and stability of the proposed method.

This contribution is based on recent joint work [1] with Sudarshan K. Kenettinkara (IIT Guwahati, India), Ricardo Ruiz-Baier (Oxford University, UK), and Héctor Torres (Universidad de La Serena, Chile).

References

[1] R. Bürger, S.K. Kenettinkara, R. Ruiz-Baier, and H. Torres. Non-conforming/DG coupled schemes for multicomponent viscous flow in porous media with adsorption. Preprint 2017-08, Centro de Investigación en Ingeniera Matemática, Universidad de Concepción; submitted.

[2] R. Bürger, S. Kumar, S.K. Kenettinkara, and R. Ruiz-Baier. Discontinuous approximation of viscous two-phase flow in heterogeneous porous media. *J. Comput. Phys.*, 321:126–150, 2016.

[3] B. Cockburn, S. Hou, and C.-W. Shu. The Runge-Kutta local projection discontinuous Galerkin finite element method for conservation laws. IV. The multidimensional case. *Math. Comp.*, 54:545–581, 1990.

[4] J. Könnö and R. Stenberg. $H(\mathrm{div})-$conforming finite elements for the Brinkman problem. *Math. Models Methods Appl. Sci.*, 21:2227–2248, 2011.

[5] D. Rodriguez, L. Romero-Zerón, and B. Wei. Oil displacement mechanisms of viscoelastic polymers in enhanced oil recovery (EOR): a review. *J. Petrol. Explor. Prod. Technol.*, 4:113–121, 2014.

[6] K. Sudarshan Kumar, C. Praveen, and G. D. Veerappa Gowda. A finite volume method for a two-phase multicomponent polymer flooding. it J. Comput. Phys., 275:667–695, 2014.

CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción
E-mail address: rburger@ing-mat.udec.cl