Seminario de Análisis Numérico y Modelación Matemática

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Augmented mixed finite element methods for nonlinear problems in continuum mechanics

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Resumen

In this talk we introduce and analyze new augmented mixed finite element methods allowing much more flexibility in the choice of the corresponding finite dimensional subspaces for a class of nonlinear problems arising in continuum mechanics. We first deal with a problem in hyperelasticity for which the incorporation of the strain tensor as an auxiliary unknown, together with the usual stress-displacement-rotation approach employed in linear elasticity, yields a nonlinear twofold saddle point operator equation as the resulting weak formulation. Then, we consider the augmented schemes obtained by adding consistent Galerkin-type terms arising first from the constitutive equation, and then from the equilibrium equation and the relations defining the rotation in terms of the displacement and the strain tensor as independent unknown, all of them multiplied by suitably chosen stabilization parameters. As a second problem, we study the coupling of quasi-Newtonian fluids and porous media, where the flows are governed by a class of nonlinear Stokes and linear Darcy equations, respectively, and the transmission conditions are given by mass conservation, balance of normal forces, and the Beavers-Joseph-Saffman law. We apply dual-mixed formulations in both domains, and, in order to handle the nonlinearity involved in the Stokes region, we set the strain and vorticity tensors as auxiliary unknowns. In turn, since the transmission conditions become essential, they are imposed weakly, which yields the introduction of the traces of the porous media pressure and the fluid velocity as the associated Lagrange multipliers. Moreover, in order to facilitate the analysis, we augment the formulation in the fluid by incorporating a redundant Galerkin-type term arising from the quasi-Newtonian constitutive law multiplied by a suitable stabilization parameter. Next, we apply classical results on the solvability analysis of nonlinear saddle point and strongly monotone operator equations, and also a new generalization of the Babuška-Brezzi theory, to show the well-posedness of all the resulting continuous and discrete formulations and to derive the corresponding a-priori error estimates. This certainly includes the previous derivation of feasible finite element subspaces for all the unknowns. In addition, we employ classical approaches, which include linearization techniques, Clément's interpolator and Helmholtz's decomposition, together with known efficiency estimates, to derive a reliable and efficient residual-based a posteriori error estimator for each one of the discrete schemes of both problems. Finally, several numerical results confirming the good performance of the augmented mixed finite element methods and the properties of the a posteriori error estimators, and illustrating the capability of the corresponding adaptive algorithms to identify the singular regions of the solutions, are reported.

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