B O O K O F

A B S T R A C T S

Talca Numérica I

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Preface

Talca Numérica I, *Tenth Meeting on Numerical Analysis of Partial Differential Equations*, is part of a series of traditional scientific meetings held in different cities in Chile.

Brief history. The idea of organizing this type of series was brought by Dr. Gabriel Gatica (Center for Research in Mathematical Engineering (CI^2MA) of the Universidad de Concepción) in 2004, based on the numerical analysis event entitled *Zaragoza Numérica*, created by Dr. Francisco-Javier Sayas in 2003. The first event in Chile, *Valparaíso Numérico I*, was held at the Universidad Técnica Federico Santa María in January 2005. The second event was also held in Valparaíso, and then the series continued in Santiago and La Serena.

In this opportunity, **Talca Numérica I** will be hosted and co-organized by the Department of Mathematics, Physics and Statistics of the Faculty of the Basic Sciences of the Universidad Católica del Maule, Talca.

The main goal is to present recent developments in Numerical Analysis of Partial Differential Equations, for which plenary lectures and contributed talks have been organized. **The present volume contains the abstracts of all contributions, which are placed following the order of the presentations in the conference programme.**

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Plenary talks

POD-ROM methods: from a finite set of snapshots to continuous parametrized approximations

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Summary. In this talk we study the discretization of time-dependent partial differential equations (PDEs) by proper orthogonal decomposition reduced order models (POD-ROMs). Most of the analysis in the literature has been performed on fully-discrete methods using first order methods in time. Our aim is to show which kind of error bounds can be obtained using any time integrator, both in the full order model (FOM), applied to compute the snapshots, and in the POD-ROM method. To this end, we analyze the continuous-in-time case for both the FOM and POD-ROM methods, although the POD basis is obtained from snapshots taken at a discrete (i.e., not continuous) set times. Optimal pointwise-in-time error bounds between the FOM and the POD-ROM solutions are proved for the $L^2(\Omega)$ norm of the error for a semilinear reaction-diffusion model problem. The dependency of the errors on the distance in time between two consecutive snapshots and on the tail of the POD eigenvalues is tracked. Our detailed analysis allows to show that, in some situations, a small number of snapshots in a given time interval might be sufficient to accurately approximate the solution in the full interval. Numerical studies support the error analysis.

In the second part of the talk we extend the previous results to parametric time- dependent partial differential equations (PDEs). Many papers in the literature consider reduced order models for parametric equations but a complete error analysis of the methods is still a challenge. In a first approach we consider a model problem depending on only one parameter (apart from time). Starting from snapshots obtained from the FOM at different times and for different values of the parameter we design a POD method that is based on a combination of different quotients both in the time and parameter variables. We obtain a priori bounds for the new method valid for any value of time in a given time interval and any value of the parameter in a given parameter interval. Our design of the POD method allow us to prove pointwise in time error estimates for any value of time in the given time interval, as opposed to average error bounds obtained typically in POD methods. We also show the extension of the error analysis to the case of a model depending on two parameters (apart from time). Most of the papers concerning POD methods for parametric equations are just based on the snapshots computed at different times and parameter values instead of their difference quotients. We show how the error analysis of the present paper can also cover the analysis of that case (that we call standard). Some numerical experiments compare our new approach with the standard one and support the error analysis.

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Shallow Water models for viscoplastic flows

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Summary. We present some developments on the Saint Venant equations from the Navier-Stokes-Bingham model. Specific finite volume schemes are designed to deal with the yield stress of the constitutive law. Comparisons with dam-break physical experiments will also be discussed.

Trefftz and quasi-Trefftz methods for time-harmonic wave propagation and beyond

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Summary. In the field on numerical partial differential equations (PDEs), many methods rely on standard bases, like polynomial bases, but other methods rely on problem-dependent bases. Trefftz methods rely, in broad terms, on the idea of approximating solutions to PDEs via Galerkin-type formulations discretized with basis functions solving exactly the governing PDE locally, making explicit use of information about the ambient medium. They are of particular interest for wave propagation problems, for example with plane wave bases with the wave number depending on the propagation medium. The presentation will start with an introduction to Trefftz methods. However, in general, for problems modeled by PDEs with variable coefficients, no exact solutions are available. Hence quasi-Trefftz methods have been introduced to address this problem: they rely not on exact solutions to the PDE but instead on high order approximate solutions constructed locally. We will discuss some of the fundamental properties of these numerical methods for problems governed by the Helmholtz equation. Finally we highlight recent developments for time-harmonic electromagnetic wave propagation and other problems governed by vector-valued equations.

Numerical methods for fractional diffusion

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Summary. The aim of the talk is to introduce two nonequivalent definitions of fractional diffusion in bounded domains, to discuss basic notions of regularity of solutions of linear elliptic problems involving these definitions, to present elementary numerical methods, and to show how regularity estimates and intrinsic properties of the proposed methods allow to obtain error estimates.

Exploring Traces versus Bubbles in the Design of Multiscale H(div)-conforming Finite Elements

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Summary. The importance of H(div)-conforming approximations is well recognized for conservative mixed formulations of multiphysics systems. The adoption of divergence-compatible finite element pairs gives rise to robust, stable and conservative approximations. For their construction, it is necessary to keep a careful track of the trace (normal component) over element interfaces and of bubble components (having vanishing traces) of the vector functions. In this talk we describe how these attributes can be explored, both with regard to the generality of mesh geometry and of the adopted local polynomial approximations.

For instance, one idea is to apply local bubble enrichment while keeping the traces at coarser resolution. This technique is crucial for hp-adaptivity or for a better resolution of refined details in multiscale problems. It can also be applied for potential or divergence accuracy improvement in the presence of non-affine elements. Illustrations shall be presented in the contexts of fluid flows and elasticity models.

Efficient Physics-Preserved Neural Network (P^2NN) Methods for Interface Problems

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Summary. Computer simulations of interface phenomena, such as interface singularities, thin transitional interior or boundary layers, and discontinuities, remain a significant challenge in computational mathematics.

This talk will first use simple examples to demonstrate that ReLU neural networks can accurately approximate discontinuous and non-smooth functions with degrees of freedom several orders of magnitude lower than those required by finite element methods on quasi-uniform mesh.

While some existing neural network-based approaches, such as Physics-Informed Neural Networks (PINNs), attempt to incorporate physical principles, they often fail to fully preserve the underlying physics. In contrast, this talk will discuss and introduce fundamentally different approaches: Physics-Preserved Neural Networks (P^2NN) methods, which rigorously enforce physical laws at the discrete level.

A major computational challenge associated with ReLU neural networks is the inherently non-convex optimization problem they produce. This talk will conclude with a discussion of our latest advancements in overcoming this critical issue, paving the way for more efficient, robust, and physically faithful neural network-based simulations.

Contributed talks

How much information can we recover from piecewise polynomial approximations?

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Summary. It is well known that the L^2 best approximation $\Pi^p_{\mathscr{T}}$ onto $\mathscr{T}$-piecewise polynomials of degree $\leq p \in \mathbb{N}_0$ satisfies

$$\|u - \Pi^p_{\mathscr{T}} u\|_{L^2(\Omega)} \lesssim h^{p+1} \|D^{p+1} u\|_{L^2(\Omega)}$$

and this result is sharp. Here, $\mathscr{T}$ is a regular simplicial mesh of the Lipschtiz domain $\Omega \subseteq \mathbb{R}^d$ ($d = 1,2,3$). In this talk I present recent results from [1] where we show how to recover approximations from $\mathscr{T}$-piecewise polynomial approximation that converge at higher rates. We do this by introducing a family of quasi-interpolation operators J^p that satisfy $J^p = J^p \circ \Pi^p_{\mathscr{T}}$, i.e., the operators only "see" piecewise polynomials of degree p, and that map into the space of piecewise polynomials of degree $p+1$. We prove for $d = 1,2$ that these operators satisfy

$$\|u - J^p u\|_{L^2(\Omega)} \lesssim h^{p+2} \|D^{p+2} u\|_{L^2(\Omega)}.$$

This means that $J^p u$ converges at a higher rate than the best approximation $\Pi^p_{\mathscr{T}} u$ although both use the same information. While the latter result is of purely theoretic nature, it can be exploited to enhance accuracy of, e.g., finite element solutions u_h. Indeed, many methods including mixed finite element methods, (hybridizable) discontinuous (Petrov–)Galerkin methods satisfy a *supercloseness* property of the form (or similar)

$$\|\Pi^p_{\mathscr{T}}(u - u_h)\|_{L^2(\Omega)} = \mathscr{O}(h^{p+2}).$$

Combining this observation with our operator we prove that

$$\|u - J^p u_h\|_{L^2(\Omega)} = \mathscr{O}(h^{p+2}), \qquad \text{although } \|u - u_h\|_{L^2(\Omega)} = \mathscr{O}(h^{p+1}).$$

While popular postprocessing like [2] achieve the same order of convergence and can be computed more efficiently, our approach extends to more general situations where, e.g., discrete gradient approximations are not directly accesible.

A second family of operators is introduced that is based on piecewise constant approximations and for $d = 1$ we establish that an arbitrary (but fixed) order of convergence can be achieved under mild assumptions on the mesh. Throughout the talk we present numerical examples.

Our main results are based on highly technical results that analyze the intersection of orthogonal polynomials on patches [3].

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Numerical analysis of a conforming finite element method for a reverse osmosis model

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Summary. This project aims to advance seawater desalination by developing efficient numerical techniques to optimize system performance. Focusing on the Reverse Osmosis (RO) process, commonly used in the world. RO desalination involves complex mathematical challenges, including coupled equations such as the Navier-Stokes equation and a diffusion-reaction equation for salt concentration. The study addresses the nonlinear interactions between fluid flow and concentration, especially at membrane boundaries, to create dynamic simulations that enhance system efficiency.

We first perform a continuous-level analysis of the coupled system to establish existence and uniqueness of solutions via a fixed-point argument, ensuring the well-posedness of the model. At the discrete level, we design a conforming finite element scheme that preserves the coupling structure and proves to be well-posed as well. Convergence tests are carried out using $\mathbb{P}_2-\mathbb{P}_1-\mathbb{P}_2$, $\mathbb{P}_3-\mathbb{P}_2-\mathbb{P}_3$ and $\mathbb{P}_{1,b}-\mathbb{P}_1-\mathbb{P}_1$ elements for velocity, pressure, and concentration, respectively, within the FreeFem++ environment. Preliminary computational simulations illustrate the formation of concentration polarization layers, and show how membrane efficiency is influenced by flow regimes and parameter choices. Sensitivity analyses with respect to inlet velocity, solute diffusivity, and membrane permeability further confirm that optimized configurations significantly reduce salt passage and enhance overall desalination performance.

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New fully mixed finite element methods for the coupled convective Brinkman-Forchheimer and nonlinear transport equations

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Summary. We introduce and analyze new Banach spaces-based fully-mixed finite element methods for the convective Brinkman–Forchheimer equations coupled with a nonlinear transport phenomenon. Our approach is based on the incorporation of the fluid velocity gradient, the incomplete nonlinear fluid pseudostress, the concentration gradient, and the total (diffusive plus advective) flux for the concentration, as auxiliary variables, which, along with the velocity and concentration themselves, constitute the set of unknowns of the model. The resulting mixed variational formulation can be written as two coupled nonlinear saddle point systems, which are then reformulated as an equivalent fixed-point equation defined in terms of the operators solving the corresponding decoupled problems. An analogue approach is utilized for the associated Galerkin scheme. In this way, the Babuška–Brezzi theory, some abstract results on monotone operators, and the classical Banach fixed-point theorem are employed to establish the well-posedness of both the continuous and discrete schemes. In particular, for each integer $k \geq 0$, vector and tensor Raviart–Thomas subspaces of order k for the pseudostress and the total flux, respectively, as well as piecewise polynomial subspaces of degree $\leq k$ for the velocity, the concentration, and their respective gradients, yield stable Galerkin schemes. Optimal a priori error estimates along with the corresponding rates of convergence are also established. Finally, several numerical experiments confirming the latter and illustrating the good performance of the method in 2D and 3D, are reported.

Key words: convective Brinkman–Forchheimer equations, nonlinear transport, pseudostress-velocity formulation, fixed point theory, mixed finite elements

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Banach space mixed formulations for the convection-diffusion-reaction system based upon Brinkman–Forchheimer equations

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Summary. We introduce and analyze new mixed formulations for coupling the Brinkman–Forchheimer equations with a convection-diffusion-reaction phenomenon within a Banach space-based framework. Specifically, for the Brinkman–Forchheimer equations, we consider a pseudostress-velocity mixed formulation, while for the convection-diffusion-reaction equations, we analyze two distinct approaches. First, we formulate the coupled problem using a mixed-primal strategy. Then, we reformulate the convection-diffusion-reaction part by introducing the pseudodiffusion vector as an additional unknown, leading to a fully mixed formulation of the coupling. In the mixed-primal approach, the Dirichlet boundary condition for the concentration is enforced through a suitable Lagrange multiplier. In contrast, the fully mixed approach avoids this requirement but introduces an additional and more restrictive assumption on the data. We establish the well-posedness of both formulations using a fixed-point strategy and prove the well-posedness of the uncoupled problems via the Babuška–Brezzi theory and the Banach–Nečas–Babuška theorem. Additionally, we provide a discrete analysis for both approaches under specific assumptions on arbitrary finite element spaces. In particular, by using Raviart–Thomas spaces and discontinuous polynomial spaces that satisfy the corresponding hypotheses, optimal theoretical convergence orders are achieved. Finally, we illustrate the theoretical results through several numerical examples, comparing both approaches and testing the associated data assumptions.

Key words: Brinkman–Forchheimer equations, convection-diffusion-reaction equations, pseudostress-velocity formulation, fixed point theory, mixed finite element methods, *a priori* error analysis

Mathematics subject classifications (2020): 65N30, 65N12, 65N15, 35Q79, 80A19, 76R50, 76D07

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A skew-symmetric-based mixed FEM for stationary MHD flows in highly porous media

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Summary. We propose and analyze a new mixed variational formulation for the coupling of the convective Brinkman–Forchheimer and Maxwell equations for stationary magnetohydrodynamic flows in highly porous media. Besides the velocity, magnetic field, and a Lagrange multiplier associated with the divergence-free condition of the magnetic field, our approach introduces a convenient translation of the velocity gradient and the pseudostress tensor as additional unknowns. Consequently, we obtain a five-field mixed variational formulation within a Banach space framework, where the aforementioned variables are the main unknowns of the system, exploiting the skew-symmetric property of one of the involved operators. The resulting mixed scheme is then equivalently written as a fixed-point equation, allowing the application of the well-known Banach theorem, combined with classical results on nonlinear monotone operators and a sufficiently small data assumption, to prove the unique solvability of the continuous and discrete systems. In particular, the analysis of the discrete scheme requires a quasi-uniformity assumption on the mesh. The finite element discretization involves Raviart–Thomas elements of order $k \geq 0$ for the pseudostress tensor, discontinuous piecewise polynomial elements of degree k for the velocity and the velocity gradient translation, Nédélec elements of degree k for the magnetic field, and continuous piecewise polynomial elements of degree $k+1$ for the Lagrange multiplier. We establish stability, convergence, and optimal *a priori* error estimates for the corresponding Galerkin scheme. Theoretical results are illustrated by numerical tests.

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Ritz–Uzawa Neural Networks for Solving Variational Problems

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Summary. We present RUNNs (Ritz–Uzawa Neural Networks), a framework that combines the classical Uzawa iterative method with neural networks to solve variational problems. By interpreting Uzawa's residual and correction steps as neural network optimizations, we propose three algorithmic approaches to update the solution iteratively. The method generalizes classical Ritz and PINN techniques and adapts naturally to ultraweak formulations. We demonstrate that, under appropriate choices, convergence is guaranteed. We provide numerical examples in 1D for the advection problem and the Poisson problem, which confirms the convergence of the iterative method.

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A monotone operator theory for the generalized porous media equation in pressure-mixed form

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Summary. The Generalized Porous Media Equation (GPME) describes the fluid flow within a porous continuum according to general (nonlinear) pressure laws. However, when used to describe fluid within poroelastic media, it has been observed to yield an inconsistent primal formulation. In this context, mixed formulations have been shown to be an effective strategy to avoid these difficulties. In this work we start by developing a general convergence theory for hemicontinuous, coercive, and monotone operators, i.e. operators satisfying the hypotheses of the Minty-Browder theorem. This theory includes *a-priori* estimates, Galerkin orthogonality, and a Ceá estimate as required to devise Finite Elements (FE) schemes. We leverage this theory to analyze the GPME under mild regularity assumptions of the pressure law, and propose robust FE spaces that yield optimal convergence rates. In addition, we further propose a quasi-Newton method that avoids Jacobian reassembly, and prove its convergence, together with an optimal preconditioner for the quasi-Newton Jacobian matrix. We validate our theoretical results with adequate numerical tests.

New twofold saddle-point formulations for Biot poroelasticity with porosity-dependent permeability

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Summary. We propose four-field and five-field Hu–Washizu-type mixed formulations for nonlinear poroelasticity – a coupled fluid diffusion and solid deformation process – considering that the permeability depends on a linear combination between fluid pressure and dilation. As the determination of the physical strains is necessary, the first formulation is written in terms of the primal unknowns of solid displacement and pore fluid pressure as well as the poroelastic stress and the infinitesimal strain, and it considers strongly symmetric Cauchy stresses. The second formulation imposes stress symmetry in a weak sense and it requires the additional unknown of solid rotation tensor. We study the unique solvability of the problem using the Banach fixed-point theory, properties of twofold saddle-point problems, and the Banach–Nečas–Babuška theory. We propose monolithic Galerkin discretisations based on conforming Arnold–Winther for poroelastic stress and displacement, and either PEERS or Arnold–Falk–Winther finite element families for the stress-displacement-rotation field variables. The wellposedness of the discrete problem is established as well, and we show a priori error estimates in the natural norms. Some numerical examples are provided to confirm the rates of convergence predicted by the theory, and we also illustrate the use of the formulation in some typical tests in Biot poroelasticity.

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FEM-BEM coupling with OSRC regularisation for high-frequency acoustic wave transmission

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Summary. The numerical simulation of high-frequency acoustic waves is of great importance in several engineering applications. Of special interest to this talk is therapeutic ultrasound. For example, focused ultrasound waves can locally treat malign tissue or modulate brain activity. Computational models aid in improving safety guidelines and patient-specific planning of ultrasound-based treatments. For these simulations to be effective, they must be stable and efficient at high frequencies, model unbounded domains, and consider local heterogeneous materials.

The finite element method (FEM) and the boundary element method (BEM) can numerically solve the Helmholtz system for acoustic wave propagation. When an object with heterogeneous wave speed or density is embedded in an unbounded exterior medium, the coupled FEM-BEM algorithm promises to combine the strengths of each technique. The FEM handles the heterogeneous regions while the BEM models the homogeneous exterior. Even though standard FEM-BEM algorithms are effective, they do require stabilisation at resonance frequencies. One such approach is to add a regularisation term to the system of equations. This algorithm is stable at all frequencies but also brings higher computational costs. This study proposes a regulariser based on the on-surface radiation conditions (OSRC). The OSRC operators are also used to precondition the boundary integral operators and combined with incomplete LU factorisations for the volumetric weak formulation. The proposed preconditioning strategy improves the convergence of iterative linear solvers significantly, especially at higher frequencies.

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IPDG methods for the nearly incompressible elasticity eigenvalue problem with heterogeneous media

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Summary. In this talk we discuss a family of interior penalty discontinuous Galerkin methods for solving the Herrmann formulation of the linear elasticity eigenvalue problem in heterogeneous media. By employing a weighted Lamé coefficient norm within the framework of non-compact operators theory, we prove convergence of both continuous and discrete eigenvalue problems as the mesh size approaches zero, independently of the Lamé constants. Additionally, we conduct an a posteriori analysis and propose a reliable and efficient estimator. Our theoretical findings are supported by numerical experiments.

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A residual-based a posteriori error estimator for a nonisothermal fluid-membrane interaction problem

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Summary. In this work we propose a residual-based a posteriori error estimator for a conforming discretization of a nonisothermal Navier–Stokes/Darcy coupled system in two and three dimensions recently introduced in [2]. More precisely, we introduce a reliable and efficient a posteriori error estimator the coupled system studied in [2] where the primal formulation of the convection-diffusion equation for the temperature, is coupled with the velocity-pressure formulation of the Navier–Stokes problem in the free-fluid region, and the dual-mixed velocity-pressure formulation of the Darcy equation in the porous medium domain. The latter, suggests the introduction of a Lagrange multiplier representing the Darcy pressure on the interface. The Galerkin scheme introduced in [2] considers the Bernardi–Raugel and Raviart–Thomas elements for the velocities, piecewise constant elements for pressures, continuous piecewise linear functions for temperatures, and continuous piecewise linear functions for the Lagrange multiplier on a partition of the interface.

Employing standard arguments such as global inf-sup conditions, suitable Helmholtz's decompositions and the local approximation properties of the Raviart–Thomas and Clément interpolation operators, we derive the a posteriori error estimator andf prove its reliability. In turn, inverse inequalities, usual localization techniques of bubble functions and known results from previous works, are employed to prove the local efficiency of the proposed error estimator. Finally, we propose an adaptive algorithm based on the local and computable error indicators of the a posteriori error estimator and provide several numerical examples to illustrate its performance and effectiveness.

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A hybridizable discontinuous Galerkin method for dissimilar meshes

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Summary. We propose and analyze a hybridizable discontinuous Galerkin (HDG) method for dissimilar meshes. This type of meshes are common, for instance, when different parts of the domains are triangulated independently, which may generate gaps or overlaps between these triangulations. The method considers an HDG discretization on separate meshes and "tie" them together through appropriate transmission conditions. These transmission conditions are based upon transferring the numerical flux from the first mesh to the second one, and the numerical trace from the second mesh to the first one. Stability and error analysis are shown, where the size of the gap is explicitly written in the estimates. We also present numerical results to validate the theory.

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Coupled Boundary-Volume Integral Equations for Wave Propagation

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Summary. We study frequency domain acoustic and electromagnetic scattering problems. In particular, we are interested in solving the transmission problem at a bounded, penetrable and inhomogeneous obstacle $\Omega \subset \mathbb{R}^d$, $d = 2,3$.
We derive a new representation formula when constant reference coefficients are given for the interior domain. The resulting integral representation contains the usual layer potentials for acoustic and electromagnetic problems, but also volume potentials on Ω. Then, it is possible to follow a single-trace approach to obtain boundary integral equations perturbed by traces of compact volume integral operators with weakly singular kernels.
The coupled boundary and volume integral equations are discretized with a Galerkin approach with standard finite elements on the boundary and in the volume. We study the well-posedness of continuous and discrete formulations. Compression techniques and special quadrature rules for singular integrands are required for an efficient and accurate method. Numerical experiments provide evidence that our new formulation enjoys promising properties.

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Parameter-robust preconditioning for hybridizable symmetric discretizations

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Summary. Hybridizable discretizations allow for the elimination of local degrees-of-freedom leading to reduced linear systems. In this paper, we determine and analyse an approach to construct parameter-robust preconditioners for these reduced systems. Using the framework of Mardal and Winther (Numer. Linear Algebra Appl., 18(1):1–40, 2011) we first determine a parameter-robust preconditioner for the full system. We then eliminate the local degrees-of-freedom of this preconditioner to obtain a preconditioner for the reduced system. However, not all reduced preconditioners obtained in this way are automatically robust. We therefore present conditions that must be satisfied for the reduced preconditioner to be robust. To demonstrate our approach, we determine preconditioners for the reduced systems obtained from hybridizable discretizations of the Darcy and Stokes equations. Our analysis is verified by numerical examples in two and three dimensions.

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Reduced Basis Model for the Elastic Wave Scattering by Multiple Cracks

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Summary. In this work, we study elastic wave scattering by multiple cracks in two space dimensions. Our goal is to construct a reduced-order model for the shape-to-solution map without solving the full problem. To this end, we build a reduced space based on the solution of a single-crack problem and demonstrate that, under certain assumptions on the shape perturbations, this space can effectively be used to discretize the multiple-crack problem.

Building on previous results, we prove that the shape-to-solution map is analytic, which in turn allows us to establish algebraic, yet dimension-independent, convergence rates for the reduced basis approximation. We present a series of numerical experiments illustrating the accuracy, efficiency, and —most importantly— the speed-up achieved by our method compared to traditional discretization techniques.

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Minimum-residual based a posteriori error estimates for mixed finite element discretizations via superconvergent local postprocessed approximations

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Summary. We propose a methodology for constructing postprocessed approximations associated with finite element discretizations for a class of mixed formulations to solve partial differential equations involving a diffusion term through residual minimization. It combines local postprocessing techniques for the primal variable with adaptive finite element discretizations via residual minimization on discrete dual norms. Such a residual minimization procedure is performed on local postprocessing schemes, commonly used in the context of mixed finite element methods, that provide postprocessed approximations with superconvergence properties. Given the local nature of that approach, the underlying saddle point problems associated with residual minimizations can be solved with minimal computational effort. We propose and study a posteriori error estimators which incorporate the built-in residual representative associated with residual minimization schemes, residual term quantifying the mismatch between discrete fluxes, and the interelement jumps of the postprocessed solution. We present numerical experiments in two dimensions using Brezzi-Douglas-Marini elements [1] for a diffusion model problem and a hybridizable discontinuous Galerkin discretization [2] for the Helmholtz equation as inputs for our methodology. The experiments align perfectly with our key theoretical findings, suggesting that our estimates are accurate. Finally, we explore the use of our methodology starting from a variational multiscale discretization of a problem involving non-Newtonian fluids [3].

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Learning-Based Localization of Singularities in PDEs

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Summary. Singularities in the solutions of partial differential equations (PDEs) pose significant challenges for numerical solvers, particularly when their location is unknown. In this talk, we present a self-supervised learning framework for detecting singular structures using only raw, unlabeled data, such as point clouds extracted from computational meshes generated by adaptive mesh refinement (AMR) strategies, as in [1]. The method relies on a filtering pretext task to extract geometric and topological features from the data. Numerical experiments demonstrate the robustness of the approach under noise, label corruption, and various types of singularities, highlighting its potential for adaptive and data-driven PDE solvers.

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A posteriori error estimates for a bang-bang optimal control problem

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Summary. We propose and analyze a posteriori error estimates for a control-constrained optimal control problem with bang-bang solutions. We consider a solution strategy based on the variational approach, where the control variable is not discretized; no Tikhonov regularization is made. We design, for the proposed scheme, a residual-type a posteriori error estimator that can be decomposed as the sum of two individual contributions related to the discretization of the state and adjoint equations. We explore reliability and efficiency properties of the aforementioned error estimator. We illustrate the theory with numerical examples.

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A Positivity Preserving CWENO scheme for the simulation of Polydisperse Sedimentation in Inclined Channels

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Summary. In this talk, a two-dimensional model for the settling of polydisperse suspensions in inclined channels is presented. This model consists of a system of conservation laws in two dimensions modelling the concentration of particles coupled with a Stokes-type equation describing the velocity of the mixture. We propose a numerical method for approximating the solution of this problem. On the one hand, the Stokes equations are discretized via finite differences where pressure equations are approximated in the center of each control volume. In contrast, the velocity components are discretized in a staggered grid by a Marker and Cell (MAC) strategy. On the other hand, the concentration equations are approximated in a finite volume approach. The solution vector should take values in a set of physically relevant values (i.e., the components are nonnegative and sum up at most to a given maximum value). It is demonstrated that this set, the so-called invariant region, is preserved by numerical solutions produced by this scheme. To achieve this property, and motivated by [2], a pair of linear scaling limiters is applied to a high-order weighted essentially non-oscillatory (WENO) polynomial reconstruction to obtain invariant-region-preserving (IRP) high-order polynomial reconstructions. We detail the coupling algorithm and simulate some scenarios of interest.

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Conservative numerical scheme for dissipative wave equation with fractional derivative damping

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Summary. In this work, we present a numerical approach for studying the asymptotic behavior of a general wave equation with a dissipative fractional derivative term. The Caputo fractional derivative and its exponential generalization, used as models for the dissipative effect in evolution equations, have several applications, such as in thermal stresses, models of porous electrodes, relaxation vibrations, and viscoelasticity. This nonclassical derivative appears as a nonlocal term and is generally approximated using quadrature formulas, some of which can be quite accurate due to their high order, but do not produce a constant decay of energy, as occurs in the continuous case.

To address this issue and eliminate oscillations, we introduce an augmented Mbodje model [1], which we discretize while preserving energy decay. We apply our results to several vibration and elasticity models [2, 3, 4, 5].

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Global superlinear linearization schemes based on adaptive strategies for solving Richards' equation

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Summary. The Richards' equation is a nonlinear degenerate parabolic differential equation, whose numerical solutions depend on the linearization methods used to deal with the degeneracy. Those methods have two main properties: convergence (global v.s. local) and order (linear v.s. quadratic). Among the main methods, Newton's Method, the modified Picard method, and the L-scheme have one good property but not the other. Mixed schemes get the best of both properties, starting with a global linear method and following with a quadratic local scheme without a clear rule to switch from a global method to a local method.

In this work, we use two different approaches to define new global superlinear and quadratic schemes. First, we use an error-correction convex combination of classical linearization methods, a global linear method and a quadratic local method by selecting the parameter λ_k^n via an error-correction approach to get fixed point convergent sequences. Second, we use a parameter τ to adapt the time step in the general Newton-Raphson method applying to three classical linearizations and the new three error-correction linearizations. Finally, we consider a combination of the L-scheme and the τ-adaptive Newton's Method, mixing both of both methods. We first built an error-correction type-Secant scheme (ECtS) without derivatives to get a superlinear global scheme. Next, we build the convex combination of the L-scheme with three global schemes: the type-Secant scheme (ECLtS), the modified Picard scheme (ECLP), and the Newton's scheme (ECLN) to obtain global superlinear convergent schemes. For the second approach, we first apply the τ-adaptation to the classical methods (τ-Newton, τ-L-scheme, and τ-modified Picard), next we apply to the error-correction schemes (τ-AtS, τ-ALtS, τ-ALP, τ-ALN) and finally a new combination of the L-scheme with the τ-Newton adaptive method (τ-LAN).

We test the twelve new schemes with five examples given in the literature showing that they are robust and fast, including cases when the Newton's scheme does not converge. Moreover, we include an example which uses the Gardner exponential nonlinearities, showing that L- and L2-schemes are too slow as linearization techniques. Some new schemes show high performance in different examples. The τ-LAN scheme has advantages, using fewer iterations in most examples.

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Analysis of a FEM with exactly divergence-free magnetic field for the stationary MHD problem

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Summary. We propose and analyze a mixed finite element method for the stationary incompressible magneto-hydrodynamic problem providing an exactly divergence-free approximation of the magnetic field and a direct approximation of the electric field. The method is based on the introduction of the electric field as a further unknown leading to a mixed formulation where the primary magnetic variables consist of the electric and the magnetic fields, and a Lagrange multiplier included to enforce the divergence-free constraint of the magnetic field, whereas the hydrodynamic unknowns are the velocity and pressure. Then the associated Galerkin scheme can be defined by employing Nédélec and Raviart–Thomas elements of lowest order for the electric and magnetic fields, respectively, discontinuous piecewise constants for the Lagrange multiplier and any inf-sup stable pair of elements for the velocity and pressure, such as the Mini-element. The analysis of the continuous and discrete problems are carried out by means of the Banach–Nečas–Babuška theorem and the Banach fixed-point theorem, under a sufficiently small data assumption and quasi-uniformity of the mesh, the latter for the discrete scheme. Finally, we derive the corresponding Cea's estimate and provide the theoretical rate of convergence.

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A momentum and mass conservative pseudostress-based mixed finite element method for fluid flow problems

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Summary. In this talk, we present the analysis of a pseudostress-based mixed finite element method for the Stokes problem that ensures both mass and momentum conservation. Mass conservation is achieved by approximating the velocity using the lowest-order Raviart–Thomas elements, while momentum conservation is enforced through a discrete Helmholtz decomposition of the piecewise-constant vector space. We establish the well-posedness of the method and derive theoretical convergence rates, including a superconvergence result for the velocity gradient approximation. A key advantage of the proposed method is its computational efficiency, as it is slightly less expensive than the classical pseudostress-based approach, while also guaranteeing mass and momentum conservation. Additionally, we extend our analysis to the Stokes problem with mixed boundary conditions and to the Navier–Stokes problem. In addition, we present numerical experiments that confirm the theoretical results.

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Solving hyperbolic partial differential equations using CUDA

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Summary. Hyperbolic partial differential equations play a crucial role in modeling phenomena such as fluid dynamics, traffic flow, and wave interactions. Due to the complexity of these equations, numerical methods are essential for obtaining solutions, particularly with the advent of parallel computing. Graphics Processing Units (GPUs), combined with NVIDIA's CUDA programming model, have enabled significant advancements in scientific computing.

This work focuses on solving hyperbolic PDEs using CUDA to exploit the massive parallel processing power of GPUs. Specifically, we address Burgers' equation, the batch monodisperse sedimentation equation, and the polydisperse sedimentation model. Different numerical approaches are implemented, considering the architecture of NVIDIA GPUs and the parallel computing model. Numerical simulations are carried out to assess the efficiency and computational performance of each proposed method.

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Nouvelle model for fingerprint-like pattern formation based on Schnakenberg PDE equations

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Summary. Fingerprint analysis and fingerprint identification has been the most use tool for human identification. Until this day, various models explaining how this are formed, being the buckling of thin layers model the most accepted one for its origin [1, 2], and some intents have had good results on the framework of synthetic fingerprint generation [4]. On this work we present a model based on reaction-diffusion equations with cross-diffusion parameters for Schnakenberg interaction terms [3] which can simulate the different structures that can be found on fingerprints. The model is capable of create fingerprint-like pattern for wide variety of pattern formation that can appear on human fingerprints on a convex domain similar to the area of the fingertip, exhibiting the different kinds of minutiae that are searched in fingerprint analysis. The patterns obtained are in good agreement with the kind of patterns found in human fingerprints exhibiting similar characteristics as real ones. This work is still in development as it is intended to give a more solid base for the mathematical approach and properties of fingerprint forensic science.

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Modeling and Simulation of the Growth and Dispersion of Plant Growth Promoting Bacteria in the Rhizosphere

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Summary. This presentation is focused on preliminary results (part of an MSc thesis) on a mathematical computational framework consisting in reaction-diffusion equations that describe the interaction between the substrate exuded by rice roots and plant growth promoting rhizobacteria (PGPR) [1] that colonize the rhizosphere. We start from a radial reformulation of the Newman and Watson model [2] and implement conservative numerical fluxes that preserve mass and respect axial symmetry [3]. Initial simulations accurately reproduce the expected substrate gradient, with a maximum at the root soil interface and a gradual decrease toward the bulk soil, and they confirm convergence to a quasi-steady regime within a few days, in agreement with the literature [2]. Thus, the coherence of the one-dimensional model has been validated. We are currently extending it to an axisymmetric two-dimensional configuration that incorporates root depth and couple the evolution of the bacterial population through substrate-dependent microbial kinetics. This extended model will allow us to study, within a single simulation environment, how soil moisture, exudation rates, and different bacterial inoculation strategies influence the system. With these advances we aim to provide a rigorous tool that supports practical recommendations for optimizing PGPR application, increasing productivity, and reducing reliance on chemical inputs in sustainable rice growing systems.

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A nonlocal coupled system: analysis and discretization

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Summary. Let $\Omega_1, \Omega_2 \subset \mathbb{R}^n (n \geq 2)$ be two bounded domains with Lipschitz boundary such that $\bar{\Omega}_1 \cap \bar{\Omega}_2 = \emptyset$. Let $s_1, s_2 \in (0,1)$ and $J \in L^1(\mathbb{R}^n)$ be a nonnegative and symmetric measurable function such that $\operatorname{supp}(J) \supset B_r(0)$, where $r > \operatorname{dist}(\Omega_1, \Omega_2)$. In this paper, we will analyze the following *nonlocal coupled system*: find $u := u_1 \chi_{\Omega_1} + u_2 \chi_{\Omega_2}$ such that

$$2\mathscr{C}_1 \int_{\Omega_2^c} \frac{u_1(x) - u_1(y)}{|x-y|^{n+2s_1}} \mathrm{d}y + \int_{\Omega_2} J(x-y)\,(u_1(x) - u_2(y))\,\mathrm{d}y = f_1(x), \quad x \in \Omega_1,$$

$$2\mathscr{C}_2 \int_{\Omega_1^c} \frac{u_2(x) - u_2(y)}{|x-y|^{n+2s_2}} \mathrm{d}y + \int_{\Omega_1} J(x-y)\,(u_2(x) - u_1(y))\,\mathrm{d}y = f_2(x), \quad x \in \Omega_2.$$

This system models long-range interactions between spatially separated regions, with both equations involving fractional-type diffusion and nonlocal coupling through the kernel J. The support condition for J ensures that the interaction region covers the gap between domains so that the model can fully account for long-range effects.

Nonlocal models are particularly effective for describing complex behaviors, such as pollutant transport [4] or crack formation and propagation [3]. In many contexts, nonlocal effects are restricted to specific parts of the domain, leading to hybrid local/nonlocal models [1]. To the best of our knowledge, this is the first work to address such a purely nonlocal coupling. Our results provide new theoretical and numerical tools for the simulation of complex systems governed by long-range interactions.

In this work, we establish well-posedness, including existence, uniqueness, and regularity of solutions. We also propose a finite element discretization with a priori error estimates, and introduce a Schwarz-type alternating iterative method [2] for both the continuous and discrete formulations, proving its convergence.

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Poster Session

A Hybrid Variational Formulation for Robust VPINNs with Localized Norms

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Summary. In recent years, machine learning has gained recognition as an effective method for approximating solutions to partial differential equations (PDEs). This approach involves training neural networks using both data and numerical constraints. Physics-Informed Neural Networks (PINNs) [1] are employed to approximate solutions of PDEs incorporating the strong form of the equations into the function. Variational Physics-Informed Neural Networks (VPINNs) [2] is a Petrov-Galerkin-type version of PINNs, based on the use of (deep) neural networks as trial space and finite dimensional space as the test space. A more recent development is the introduction of Robust Variational Physics-Informed Neural Networks (RVPINNs) [3], which build upon VPINNs to improve numerical stability and robustness. This is achieved by minimizing a discrete dual norm of the residual, offering stronger error control and better performance. However, a significant computational challenge appears from the need to invert a Gram matrix, which can be particularly costly if the variational formulation and the test space are not carefully selected. To mitigate this issue, we present RVPINNs within the framework of variational formulations that allow the norm to be localized. Building on this, we propose a hybrid variational formulation that preserves the intrinsic robustness of RVPINNs while reducing their computational cost.

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A Discontinuous Galerkin Method for the Stationary Boussinesq System

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Summary. In this work we present and analyze a finite element scheme yielding discontinuous Galerkin approximations to the solutions of the stationary Boussinesq system for the simulation of non-isothermal flow phenomena. The model consists of a Navier–Stokes-type system, describing the velocity and the pressure of the fluid, coupled to an advection-diffusion equation for the temperature. The proposed numerical scheme is based on the standard interior penalty technique and an upwind approach for the nonlinear convective terms and employs the divergence-conforming Brezzi–Douglas–Marini (BDM) elements of order k for the velocity, discontinuous elements of order $k-1$ for the pressure and discontinuous elements of order k for the temperature. Existence and uniqueness results are shown and stated rigorously for both the continuous problem and the discrete scheme, and optimal a priori error estimates are also derived. Numerical examples back up the theoretical expected convergence rates as well as the performance of the proposed technique.

Residual minimization techniques for solving nonlinear Partial Differential Equations in Banach spaces.

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Summary. This work focuses on the theoretical and numerical study of a method based on residual minimization in dual norms to solve nonlinear Partial Differential Equations defined in Banach spaces. The process involves a mixed formulation derived from the problem of minimizing the residual in the dual norm, along with the introduction of a representative of this residual, defined in terms of the duality map of the test space [2]. Building on [1], we propose an algorithm preserving the symmetry of the corresponding linearized saddle-point formulation, which guarantees solvability at every step of the nonlinear solver.

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A conservative HDG method for the coupled Navier-Stokes and Transport equations

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Summary. We consider the coupled Navier-Stokes/transport equations, with nonlinear conditions across a semipermeable membrane. This system of partial differential equations arises from reverse osmosis modeling in water desalination processes. To solve the Navier-Stokes equations, we use a conservative hybridizable discontinuous Galerkin (HDG) scheme in primal form. For the transport equation, we also employ an HDG method in primal form. The main advantage of our formulation, in addition to the conservative property, is the fact that the size of the global linear system to solve is smaller compared to non-hybrid schemes. We show the scheme is well-posed under smallness assumptions on the data, and show numerical experiments validating the theory.

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Weighted Schur and Markov Inequalities on the d-simplex

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Summary. We aim to prove the weighted Markov inequality on the d-simplex:

$$\left(\forall p \in \Pi_N^d\right) \ ||\nabla p||_{\mathrm{W}\gamma} \leq C(\gamma,d)N||p||_{\mathrm{W}\gamma} \tag{1}$$

where $C(\gamma,d) > 0$, $\gamma \in (-1,\infty)^{d+1}$, and $\mathrm{W}\gamma$ is a weight function on the d-simplex defined by

$$\mathrm{W}\gamma(x) := x_1^{\gamma_1} \ldots x_d^{\gamma_d} (1 - x_1 - \cdots - x_d)^{\gamma_{d+1}}.$$

To prove (1), we need to establish a type of weighted Schur inequality and a Bernstein inequality (see [1]). The proof of the weighted Markov inequality is based on an induction argument on the dimension, which is carried out by means of a nonlinear map that relates the d-simplex to the tensor product of the $(d-1)$-simplex and the interval $(-1,1)$. The inequality (1) is crucial for studying the approximation properties of weighted L^2-orthogonal projectors onto spaces of polynomials of bounded degree in the d-simplex (see [2] and [3]).

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A mixed formulation for the Landau-de Gennes equations in liquid crystals modelling

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Summary. We study the minimization of the H^1 seminorm for a second-order tensor field, subject to symmetry and zero-trace constrains and a given external force, along with Dirichlet boundary conditions. The problem was motivated by the modeling of cardiac muscle fiber orientation, where the goal is to reconstruct a smooth tensor field that reflects the spatial organization of myocardial fibers and that also respects the structural constraints of the cardiac tissue. The mathematical framework is given by the Landau–de Gennes theory for nematic liquid crystals. We handle the constraints through Lagrange multiplies, which results in a nonlinear mixed formulation. A standard Lagrangian approach through L^2 products does not yield a well-posed problem. This has motivated the use of H^1 products for penalization instead of L^2, leading to a well-posed saddle-point problem. We prove well-posedness in both continuous and discrete settings with a linear external force and derive an optimal a priori error estimate in H^1 norm, ilustrated through numerical experiments that demonstrate the optimal convergence of the method.

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