ERROR ESTIMATES FOR PARTITION OF UNITY FINITE ELEMENT SOLUTIONS OF THE HELMHOLTZ EQUATION

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Abstract. The Helmholtz equation is widely used as the reference model in time-harmonic acoustic propagation problems. At middle and high frequency regime, its numerical approximation, computed by a nodal Finite Element Method (FEM), differs significantly from the exact solution due to the so-called "pollution" effect [1]. So, the accuracy and reliability of the Helmholtz numerical approximations are based on pollution-free discrete methods, which should have a robust behavior with respect to the wavenumber.

The Partition of Unity Finite Element Method (PUFEM) [2] has been considered among these pollution-free methods. Computational advantages and implementation drawbacks of the PUFEM discretization have been studied numerically in [3]. Error estimates for Finite Element solutions of the unidimensional Helmholtz equation have been already studied by Babŭska and Ihlenburg in [4].

In this work, *a priori* error estimates are derived for PUFEM, where plane waves are used to modify the discretization space. The approximability of the exact solution in such discrete space is deduced from some interpolation estimates involving only exponential-type basis functions. Error estimates for PUFEM are obtained in terms of the wavenumber on the Helmholtz equation, the mesh size and an additional perturbation parameter introduced in the wavenumbers of the basis functions of the discrete PUFEM space.

References

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