FINITE ELEMENT APPROXIMATION OF ELLIPTIC PROBLEMS WITH NON-STANDARD GROWTH CONDITIONS

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ABSTRACT. In an image processing problem, the aim is to recover the real image I from an observed image ξ of the form $\xi = I + \eta$, where η is a noise. In [1], the authors introduce a model that involves the $p(x)-$Laplacian (i.e $\Delta_{p(x)} u = \operatorname{div}(|\nabla u|^{p(x)-2}\nabla u)$), for some function $p : \Omega \to [p_1, 2]$, with $p_1 > 1$. More precisely, they minimize in $W^{1,p(\cdot)}(\Omega) \cap L^2(\Omega)$ the functional

$$J(u) = \int_\Omega |\nabla u|^{p(x)} + \frac{\lambda}{2}\int_\Omega |u-\xi|^2\,dx, \tag{1}$$

where λ is a parameter and the function $p(x)$ encodes the information on the regions where the gradient is sufficiently large (at edges) and where the gradient is close to zero (in homogeneous regions). In this manner, the model avoids the *staircasing* effect still preserving the edges.

Motivated by this application, in the two dimensiona case, we study the rate of convergence of the Galerkin FEM for (1), see [3].

We prove that, if we only assume that the solution is in $W^{2,p(\cdot)}(\Omega)$ then the order is $p_1/2$. On the other hand, assuming more regularity over the solution, we obtain optimal order of convergence. In fact, in [2] we prove the $H^2(\Omega)$ regularity of the solutions.

Finally, we present some numerical experiments to compare with the theoretical results.

REFERENCES

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