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Abstract

We study a worthwhile to change model by using reference-dependent preference relations. We prove an existence result for maximal elements of the model which generalizes the well known Bronsted maximum principle and apply it to the problems of behavioral traps, satisficing change and Nash equilibrium in games with worthwhile to change payoff relations. In our study no compactness assumption is required.

1 Introduction

Theories of stability and change consider two related problems. The first problem deals with the question of static character: "should I stay" for which an anwser could be an optimal solution or an equilibrium state of a model. The second problem treats the question of dynamic character "should I go" that produces improving processes. In the latter consideration agents improve their actions with respect to a current preference and generate a new preference for next actions. Thus, preferences change with the history of past actions, experience, inertia and learning. In economics theoreticians advocate for very clever agents who optimize their daily decisions. But there is more and more empirical evidence that preferences are context and reference dependent. They forbid a global view of a situation and push agents to act locally, not globally in order to improve their state step by step. This is a central idea of change theory.

Today change problems attract attention of a big number of researchers and the list of works devoted to them is impressive. They concern almost every economic activity including among others, consumer, producer, worker, health, social, cultural activities and belief's formation. According to Schumpeter, 1934 [40], the only thing that does not change is that all things change. For Hayek, 1945 [19], economic problems arise always and only in consequence of change; while for Williamson, 1991 [52], continuous or discrete adaptation is the central economic problem. Here are some known approaches to theory of change:

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1) Lewin's theory of resistance to change (Lewin, 1951 [26]). In his work Lewin, an alter-ego of Freud in Psychology, has developed, without any mathematical modeling, a "force field analysis" which refers to any current level of performance or being as a temporary equilibrium between driving forces and restraining forces, pushing upward and downward. Driving forces or motivations to change are usually "positive, reasonable, logical, conscious and economic", while restraining forces or resistances to change are usually "negative, emotional, illogical, unconscious and social-psychologic". He suggested that "change would be easier and longer lasting if the forces against change were reduced, rather than the forces for change being increased", and developed a model of change composed of three stages: "unfreezing stage" (reducing resistances to change from the statu quo), moving stage, and refreezing stage (stabilizing after change has been made).

2) Lindblom's muddling through approach (Lindblom 1959, [27]). According to Lindblom agents improve step by step, from time to time, making limited comparisons between gains and losses, bracketing more or less their choices and actions, making small steps. In this case compensation rules between losses and gains are local, not global.

3) Simon's satisficing approach (Simon 1955,1967, 1982 [41, 43, 45]. In this approach, agents "explore enough" to "improve enough", which means that an agent, instead of optimizing over the whole space, sets a satisficing level and explores locally to improve. If he succeeds he will set a new and higher satisficing level and try again to reach a better position; otherwise he lowers his initial satisficing level and repeats his action.

4) Schumpeter's intuition concept (see Schumpeter, 1934 [40], Witt 2002, [53]). Here the entrepreneur is the agent who breaks habits and resistance to change to innovate and drive change.

5) Learning theories on the development of abilities-capabilities-competence-skills-knowledge (see the resource based theory of the firm, Mahoney, 2004 [32]). These approaches focus on the dynamics of organizations and emphasize the importance of developing inside the firm both general and specific resources and capabilities. The main point is to understand how organizations succeed to generate and protect rents (value creation, profits), and to appropriate them (value capture). In this context inertia (routines) and innovations (breaking old routines, knowledge creation and implementation, and the formation of new routines) play a major role.

Change theories do not suppose that agents always optimize. For instance, habits (for one agent, see Prendergrast et al. 2008 [36]), routines (for agents within an organization, see Nelson-Winter, 1982 [35]; Becker, 2001 [10]), norms of behavior and behavioral traps for interacting agents of a game (see Heifetz-Minelli, 2006 [21]; Ray, 2003 [37]) are present in all aspects of our individual and social life, but certainly have little to do with optimization. There are two basic concepts related to the theory of change: motivation and resistance to change. Motivation to change includes, among others, emotional, goal setting and expectancy-valence aspects (Anderson, 2007 [2], Heath-Larrick-Wu, 1999 [20], Vroom, 1964 [51], Locke-Latham, 1990 [28]). Resistance to change has its root in the psychology of group dynamics (Lewin, 1947, 1951 [25, 26]) which is considered as inertia in Management Sciences (Hannan-Freeman, 1984 [18]; Rumelt, 1995 [38]). Resistance to change incorporates both learning (exploration, capability building) and inertia aspects. To model and unify several approaches to the theory of change, Soubeyran, 2009, 2010[48, 49] has developed a theory of "worthwhile changes" where each agent uses a numerical representation of a "worthwhile to change preference" to balance his motivation and resistance to change (see also Attouch-Soubeyran, 2006, 2010 [3, 7], for more specific cases). In this context, starting from an initial state, bounded rational agents, based on their variable preferences, improve or "improve enough" (satisfice) or eventually optimize his situation, depending of the context. They eventually reach a final state, known as a behavioral trap, where they prefer to stop than to move again. The question is to know when agents, starting from an initial situation and following a path of worthwhile changes, will stop to change, reaching a behavioral trap in a finite number of steps. Behavioral traps are important empirically because they model habits, routines, norms, and equilibria which can be reached from an initial situation following a finite habituation, routinization or "learning to play Nash" process. In this paper we examine the "should I go" problem, by using reference-dependent "worthwhile to change preferences", for both isolated agents and interactive agents (a game situation). We emphasize that our aim is not only to establish conditions of existence of behavioral traps but to give also conditions for their reachability in a finite number of steps, which is one of the goal of a theory of change. An equilibrium which is not reachable in a finite number of steps has little practical contents.

The paper is organized as follows. In Section 2 we present a "worthwhile to change preference" model along with some concrete instances. In section 3 we give a construction of worthwhile to change functions in production theory. In Section 4 we establish general conditions for existence and reachability of maximal actions in a finite number of worthwhile steps for the worthwhile to change problem. The main result of this section is a generalized version of a famous theorem of Brondsted, 1974 [14] on existence of maximal elements in a uniform space. Section 5 deals with behavioral traps and conditions for their existence. In Section 6 we briefly show a link of worthwhile to change with satisficing processes. In Section 7 we consider interacting agents (games) equipped with worthwhile to change variable preferences. A short conclusion is given in the final section.

2 Worthwhile to change preferences

In this section we develop a worthwhile to change model of Soubeyran [48, 49] in the context of reference-dependent preferences. Let X denote a space of actions of an agent or a space of states of an economic system. In production theory elements of X are actions of producing some quantity of a final good of a given quality. In consumption theory elements of X represent actions of consuming bundles of goods. Let Δ be a real function on $X \times X$. Given a reference point $z \in X$ and two points $x, y \in X$, we say that an agent prefers y to x with respect to the reference point z if

$$\Delta(z, y) \geq \Delta(z, x) \tag{1}$$

and write $x \leq_z y$. The preference "$\leq_z$" is a particular case of reference-dependant preferences or variable preferences studied in Luc-Soubeyran [31]. In the change theory it is common that the reference point z coincides with x a current position of the agent. In other words, being at x the agent uses his criteria to decide whether to move to y or not. Thus, the preference will be given as

$$x \leq_x y \Leftrightarrow \Delta(x, y) \geq \Delta(x, x). \tag{2}$$

As we have already discussed in the introduction any change depends on motivation to change and resistance to change. In [48, 49], the author quantifies motivation to change and resistance to change by real functions $M(x,y)$ and $R(x,y)$ defined on $X \times X$. The worthwhile to change function is then given by

$$\Delta(x,y) = M(x,y) - R(x,y). \tag{3}$$

Intuitively it is worthwhile to change from x to y if the motivation is higher than the resistance, or equivalently $x \leq_x y \Leftrightarrow \Delta(x,y) \geq 0$. This, of course, is in concordance with (1) when $M(x,x) = R(x,x)$ that is the motivation and the resistance to stay at x are of equal value. In section 3 we give an example of a production model to understand the construction of worthwhile to change functions in a particular situation. In our paper we pay attention to the worthwhile to change preference given by (2) and (3) which are directly related to the theory of change. Here are some elementary properties of the preference given by (3):

i) The preference "$\leq_x$" is reflexive, i.e., $x \leq_x x$ for all $x \in X$. It is straightforward from (1).

ii) The preference "$\leq_x$" is not transitive in general, in the sense that $x \leq_x y$ and $y \leq_y z$ does imply $x \leq_x z$. A sufficient condition for the preference to be transitive is that a) $\Delta(x,x) = 0$ for every $x \in X$, and b) $\Delta(x,y)$ is superlinear (that is $\Delta(x,y) + \Delta(y,z) \leq \Delta(x,z)$ for all $x,y,z \in X$). The latter condition is true if the motivation to change function is superlinear and the resistance to change function is sublinear, that is for all x,y,z from X one has

$$\begin{aligned} M(x,y) + M(y,z) &\leq M(x,z) \\ R(x,y) + R(y,z) &\geq R(x,z). \end{aligned}$$

iii) The preference "$\leq_x$" is not antisymmetric in general. A sufficient condition for its anti-symmetry is that a) $R(x,y) > 0$ if and only if $x \neq y$; and b) the motivation to change function is determined by a potential function, that is $M(x,y) = \Phi(y) - \Phi(x)$ where Φ is a real function on X. Indeed, for $x,y \in X$, one has $x \leq_x y$ and $y \leq_y x$ if and only if

$$\begin{aligned} \Phi(y) - \Phi(x) &\geq R(x,y) - R(x,x) \\ \Phi(x) - \Phi(y) &\geq R(y,x) - R(y,y) \end{aligned}$$

which implies

$$R(x,y) + R(y,x) \leq 0.$$

By the hypothesis, the latter inequality holds if and only if $x = y$.

A particular case of worthwhile to change preferences is the so-called threshold preference in the theory of choice (see [1], page 33). We recall that a real function $\varepsilon(\cdot,\cdot) : X \times X \to I\!R$ is a threshold function on X if $\varepsilon(x,y) = \varepsilon(y,x) \geq 0$ and $\varepsilon(x,x) = 0$ for all $x,y \in X$. Given a utility function $u(.)$ and a threshold function $\varepsilon(\cdot,\cdot)$ on X we define a worthwhile to change preference "$\leq_x$" by

$$x \leq_x y \text{ if and only if either } x = y \text{ or } u(y) - u(x) > \varepsilon(x,y).$$

This preference corresponds to the case where the resistance function is given by

$$R(x,y) = \begin{cases} \varepsilon(x,y) & \text{if } y \neq x \\ 0 & \text{if } y = x \end{cases}$$

and the motivation function is defined by

$$M(x,y) = u(y) - u(x).$$

Notice that even in this particular case the worthwhile preference is not necessarily transitive.

3 A construction of worthwhile to change functions in production theory

To understand the construction of worthwhile to change functions let us consider a model of production (Soubeyran [48, 49]). Given an action $x \in X$ representing a process of producing an outcome (a good for instance). It is characterized by (i) an outcome φ_x from a space Φ of products; (ii) means to carry out the action x, including a list of resources $h_x \in H$ and a list of ingredients $u_x \in U$; and (iii) a transformation process $\tau_x \in \Upsilon$ and an operating system $e_x \in E$ (efforts) which mobilizes the means and transforms the ingredients into a final outcome φ_x. A productive action $x = (h_x, u_x, \tau_x, e_x, \varphi_x)$ models the fact that to produce a final outcome φ_x, one exploits resources h_x and ingredients u_x, by using a transformation process τ_x and an operating system (efforts) e_x to transform and mix ingredients. Every action generates a revenue, denoted $r(x) \in I\!R_+$, and requires a total cost. The total cost includes (i) the cost to carry out action x, denoted $c(x) \in I\!R_+$ (costs to use resources to mix and transform ingredients as well as costs of operating efforts); (ii) the maintenance cost for the capability to carry out action x, denoted $\widetilde{m}(x, \chi_x)$, where $\chi_x \in \chi(x)$ is the capacity of carrying out action x and $\chi(x)$ a set of capabilities to carry out this action. The maintenance cost at x can be defined as the infimum of the maintenance costs over the set of feasible capabilities to do action x, $m(x) = \inf \{\widetilde{m}(x, \chi_x), \chi_x \in \chi(x)\}$. An other solution is to take $m(x) = \widetilde{m}(x, \chi_x)$, for some given choice of a capability to do action $x \in X$, $\chi_x \in \chi(x)$. This means that the agent can use a decision rule $x \in X \longmapsto \chi_x \in \chi(x)$. It is important to notice that to carry out an action is not the same as to be able to do it. The capability to do an action x requires to have physical access to tangible resources (like machines, to be close to them, and to have the right to use them), and to own or acquire intangible resources to know how to activate these resources, using operating efforts, to be able to do these operating efforts, to know how to mix step by step ingredients, in which order, how long and when for each mixing operation, which succession of intermediate states of the final good to choose to follow.
The net payoff over a period is the difference between the revenue and the total cost,

$$g(x) = r(x) - c(x) - m(x).$$

All functions $r(x), c(x)$ and $m(x)$ depend on the model under consideration. For instance, if x is an action to produce a quantity φ_x of a good or a service, then, for a worker, the cost to do x is the cost of his effort $c(x) = \delta(e_x)$, the maintenance cost is the cost to recover the same capability to do that action, given that he has done it just before, and the revenue is his wage. For a producer, his revenue is given by the expression $r(x) = p_x\varphi_x - w_x - k_x$

where p_x is the unit price of the good, w_x is the wage paid to workers and k_x is the cost to use resources and ingredients. His cost $c(x)$ to do x represents costs for monitoring and selling φ_x, and his maintenance cost is the cost to be able to monitor workers and to sell the final good once again. With help of the net payoff it is now easy to define a worthwhile to change function

$$\Delta(x, y) = M(x, y) = g(y) - g(x), \tag{4}$$

where, in a static context, there is no resistance to change, that is $R(x, y) = 0$, and the motivation to change is the usual marginal gain, $M(x, y) = g(y) - g(x)$.

In a dynamic setting we do not consider actions as isolated elements but in their interrelations. Worthwhile to change functions are more complicated and cannot be expressed by a potential function of one variable such as in (4). For instance a payoff of action y after having done an action x often depends on x too, say $g(y|x)$. The worthwile to change model of production defines a so-called advantage to change function

$$A(x, y) = g(y|x) - g(x|x)$$

and considers the motivation to change function as a utility of the advantage to change

$$M(x, y) = u(A(x, y)), \tag{5}$$

where u is a strictly increasing function taking the null value at zero. If the total cost of doing y after having done x, which is a function of both x and y, is $C(x, y)$, the model defines the resistance to change function as a desutility of the cost function, that is

$$R(x, y) = d(C(x, y)), \tag{6}$$

where d is an increasing function taking the null value at zero as u does. Determination of the total cost is sometimes a difficult task in the dynamic setting. This is because in addition to the costs of performing x and y, there is a cost of passing from x to y. The most relevant part of that cost is the cost to pass from the capability of performing action x to a capability of performing action y. It depends on the path chosen to acquire, step by step, the capability χ_y of doing action y from the capability χ_x of doing action x. Let $P[\chi_x, \chi_y]$ denote the space of paths from χ_x to χ_y. Then, the cost to change capabilities is a function, denoted $\Psi(\chi_x, \chi_y, p) \geq 0$, of the following variables: a capability χ_x of doing action x, a capability χ_y of doing action y, and a path p of getting χ_y from χ_x. This cost includes inertia costs to break some habits, and learning costs to acquire new habits, costs to stop doing some old activities and costs to start doing new activities. For example for a consumer capability costs include (i) stopping costs (to decide to stop consuming some goods within the initial bundle, which break temporary habits), (ii) searching costs (to find new goods to be included in the new bundle of goods), (iii) costs to be able to continue to consume some goods, and finally (iv) starting costs (to learn how to consume new goods). In the literature capabilities are defined as abilities, skills and competence. For instance in Management Sciences the resource based theory of the firm (Barney [8]), including the evolutionary theory of the firm (Nelson-Winter [35]) considers dynamic capabilities as the abilities to change his capabilities (see the survey of Menon-Mohanty [33]). In Economics human capital and education theories (Becker [9]) consider costs of construction of skills. Sen [39] uses capabilities to built his theory of development and inequalities.

For simplification, the model supposes that there is a decision rule $x \in X \longmapsto \chi_x \in \chi(x)$ which, for each action $x \in X$, helps the agent to choose one capability among all the feasible capabilities to do this action. This is the case if there is only one capability to do an action, ie $\chi(x) = \{\chi_x\}$ for all $x \in X$. Or, one can suppose that i) there is only one capability to do each action which minimizes its maintenance costs, ie $\chi_x = \arg\min \{\widetilde{m}(x, \chi'_x), \chi'_x \in \chi(x)\}$ and, ii) $\widetilde{m}(x, \chi'_x) > \widetilde{m}(x, \chi_x)$, for all $\chi'_x \in \chi(x), \chi'_x \neq \chi'_x$. Let

$$C(x, y) = \inf \{\Psi(\chi_x, \chi_y, p) : \; p \in P[\chi_x, \chi_y]\} \geq 0. \tag{7}$$

This function on $X \times X$ represents the minimum cost to be able to do an action $y \in X$, being able to do an action x. It is not symmetric; however it has some nice properties. A first observation is that $C(x, x) = 0$, which means that the minimum cost to be able to repeat an action x is zero. This is because the maintenance cost that is exactly the cost of being able to do x, having done it just before, is already included in the payoff $g(x)$. Another important property of the minimum cost to change is the usual triangle inequality:

$$C(x, z) \leq C(x, y) + C(y, z) \tag{8}$$

for all triple actions x, y and z from X. In fact, if p is a path from capability χ_x to χ_y, and q is a path from capability χ_y to χ_z, then the joint path $q \circ p$ is a path from χ_x to χ_z. Moreover the cost to change is split into a sum

$$\Psi(\chi_x, \chi_z, q \circ p) = \Psi(\chi_x, \chi_y, p) + \Psi(\chi_y, \chi_z, q).$$

From (7) we deduce that

$$C(x, z) \leq \Psi(\chi_x, \chi_z, q \circ p) = \Psi(\chi_x, \chi_y, p) + \Psi(\chi_y, \chi_z, q)$$

for all $p \in P[\chi_x, \chi_y], q \in P[\chi_y, \chi_z]$.

Then, given x, y, z, χ_x, χ_y, and $p \in P[\chi_x, \chi_y]$, we have

$$C(x, z) - \Psi(\chi_x, \chi_y, p) \leq \Psi(\chi_y, \chi_z, q) \text{ for all } \; q \in P[\chi_y, \chi_z].$$

It follows that $C(x, z) - \Psi(\chi_x, \chi_y, p) \leq C(y, z) = \inf \{\Psi(\chi_y, \chi_z, q), q \in P[\chi_y, \chi_z]\}$.

This implies $C(x, z) - C(y, z) \leq \Psi(\chi_x, \chi_y, p)$ for all $p \in P[\chi_x, \chi_y]$.

Then, $C(x, z) - C(y, z) \leq C(x, y) = \inf \{\Psi(\chi_x, \chi_y, p), p \in P[\chi_x, \chi_y]\}$ implying (8), that is, $C(x, y)$ is a quasi-distance on X.

A more general cost to change can be obtained as follows. Define

$$C(x, y) = \sup \left\{\widetilde{C}(\chi_x, \chi_z, x, z), \chi_x \in \chi(x), \chi_z \in \chi(z)\right\},$$

where

$$\widetilde{C}(\chi_x, \chi_z, x, z) = \inf \{\psi(\chi_x, \chi_z, \gamma) : \gamma \in P[\chi_x, \chi_z]\}.$$

Then

$$\begin{aligned}
\widetilde{C}(\chi_x, \chi_z, x, z) &\leq \inf \{\psi(\chi_x, \chi_z, q \circ p) : p \in P[\chi_x, \chi_y], q \in P[\chi_y, \chi_z]\} \\
&\leq \inf \{\psi(\chi_x, \chi_y, p) + \psi(\chi_y, \chi_z, q) : p \in P[\chi_x, \chi_y], \; q \in P[\chi_y, \chi_z]\} \\
&= \inf \{\psi(\chi_x, \chi_y, p) : p \in P[\chi_x, \chi_y]\} + \inf \{\psi(\chi_y, \chi_z, q) : q \in P[\chi_y, \chi_z]\} \\
&\leq \widetilde{C}(\chi_x, \chi_y, x, y) + \widetilde{C}(\chi_y, \chi_z, y, z)
\end{aligned}$$

and yields

$$C(x, z) \leq C(x, y) + C(y, z) \;\; \forall \; x, \; y, \; z \in X.$$

A problem with this more general definition of costs to change is the increased difficulty of the agent to solve the cost to change problem. A satisficing resolution of this problem works in two steps. For given actions x and z,

i) find feasible capabilities χ_x and χ_z to be able to do actions x and z. Given a minimizing cost to change satisficing ratio $0 < \xi < 1$ and given the actions x and z, the agent can always find feasible capabilities $\chi_x \in \chi(x), \chi_z \in \chi(z)$ such that $0 \leq \widetilde{C}(\chi_x, \chi_z, x, z) \leq \xi C(x, z)$.

ii) find a way $\gamma \in P[\chi_x, \chi_z]$ to move from one capability to the other. Given the ratio $0 < \xi < 1$, and given the feasible capabilities $\chi_x \in \chi(x), \chi_z \in \chi(z)$ the agent can always find a feasible way $\gamma \in P[\chi_x, \chi_z]$ to move from χ_x to χ_z such that $0 \leq \psi(\chi_x, \chi_z, \gamma) \leq \xi\widetilde{C}(\chi_x, \chi_z, x, z)$.

Then, for a given minimizing satisficing ratio $0 < \xi < 1$, and for given actions x and z, the agent can always find feasible capabilities χ_x, χ_z to do these actions and a way γ to move from the first to the second capability such that $0 \leq \psi(\chi_x, \chi_z, \gamma) \leq \xi\widetilde{C}(\chi_x, \chi_z, x, z) \leq \xi^2 C(x, z)$.

For simplification take $R(x, z) = d(C(x, z)) = C(x, z)$ for the resistance to change function. Then, if the agent is able to find a worthwhile change from x to z such that $M(x, z) \geq R(x, z) = d(C(x, z))$, the agent can find feasible capabilities χ_x, χ_z to do these actions and a way γ to move from the first to the second such that $M(x, z) \geq C(x, z) \geq (1/\xi^2)\psi(\chi_x, \chi_z, \gamma)$.

4 Maximal actions

The aim of this section is to establish existence of actions called maximal actions, from which it is not worthwhile to pass to other actions. We need some definitions concerning reference-dependent preference relations (already introduced in Luc-Soubeyran [31]). Assume that "$\leq_x$" is a variable preference on X. An action $x_* \in X$ is said to be (ex ante) maximal if there is no action $y \in X$ such that $x_* \leq_{x_*} y$ and $y \nleq_y x_*$ where "$\nleq_y$" means negation of $\leq_y$.

Improving paths. Let $\{x_0, x_1, \ldots, x_n\}$ be a finite subset of X. We say that it forms an improving path or an upward path from x_0 to x_n if

$$x_0 \leq_{x_0} x_1 \leq_{x_1} x_2 \ldots x_{n-1} \leq_{x_{n-1}} x_n. \tag{9}$$

When $n = 1$ we say that the path is direct (i.e., $x_0 \leq_{x_0} x_n$) from x_0 to x_n. It is indirect if $n > 1$.

Acyclic preferences. The preference "$\leq_x$" is acyclic if for any improving path (9) from x_0 to x_n in X, equality $x_n = x_0$ implies $x_i = x_0$ for all $1 \leq i \leq n$. Improving paths are used to construct a transitive preference when the given preference is not transitive. Let us define the upper transitive closure of "$\leq_x$" to be a preference, denoted "$\leq^u$" in which $x \leq^u y$ if and only if there is an improving path from x to y. It is known (see Luc-Soubeyran [31, Proposition 4]) that the upper transitive closure " $\leq^u$" is a partial order on X if and only if the preference "$\leq_x$" is acyclic. A strict upper transitive closure " $<^u$" is understood as "$x <^u y$" if and only if $x \leq^u y$ and $y \nleq^u x$.

Weak consistency. We say that preference "$\leq_x$" is weak consistent if there is a direct path from x to y and an indirect path from y to x, then, there is a direct path from y to x. The concept of weak consistency has been introduced in Luc-Soubeyran [31] which generalizes some classical notions of consistency such as path consistency and Suzumura's consistency (see Bossert-Suzumura [12]). The importance of weak consistency is seen from the following fact. When the variable preference "$\leq_x$" is not acyclic, the upper transitive closure "$\leq^u$" is not anti-symmetric, hence it is not a partial order on X. However, the induced preference "$\preceq^u$" on equivalent classes of X is a partial order. If in addition "$\leq_x$" is weak consistent, then every element of a "$\preceq^u$"-maximal class is also maximal with respect to the preference "$\leq_x$" on X (see [31, Proposition 6]). Intuitively weak consistency means that if, ex ante, an agent prefers to move directly from x to y and then, following an indirect improving path, to move from y to x, ex post he will immediately regret to have directly moved from x to y, preferring, at y, to come back directly to x.

Preference-complete sets. Given an element a of X, the upper section of X at a is the set

$$S(a) = \{x \in X : \ a \leq^u x\}.$$

A subset of X is called preference-complete (or $P-$complete for short) if it has no covering of type $\{X \setminus S(x_\alpha) : \ \alpha \in I\}$ with $\{x_\alpha\}_{\alpha \in I}$ a strictly increasing net in that subset, i.e., $x_\alpha <^u x_\beta$ for $\alpha < \beta$. This hypothesis is equivalent to the following more intuitive property (see Luc-Soubeyran[31]). First, we say that an element $x_* \in X$ is called an aspiration point of a net $\{x_\alpha, \alpha \in I\}$ if $x_* \in S(x_\alpha)$ for all $\alpha \in I$. Then, a subset of X is called preference-complete if it contains an aspiration point for any strictly increasing net.

The lemma below is a key argument to obtain further results.

Lemma 4.1 *Assume that X is a Hausdorff topological space equipped with a variable preference "$\leq_x$" and that G is a real function on $X \times X$. Assume further that for every element $a \in X$ the following conditions hold:*

- (i) *$S(a)$ is a complete subspace of X;*
- (ii) *$G(a, \cdot)$ is increasing with respect to the transitive closure "$\leq^u$" and bounded above on $S(a)$;*
- (iii) *A net $\{x_\alpha\}_{\alpha \in I}$ in $S(a)$ is Cauchy if it is increasing and the real net $\{G(a, x_\alpha)\}_{\alpha \in I}$ is convergent.*

Then starting from any point $a \in X$ there is an improving path from a to a maximal element of X.

Proof. We wish to apply a recent result by Luc and Soubeyran (Theorem 9, [31]) which says that every P-complete subset of a space equipped with a weakly consistent reference-dependent preference has maximal elements. First we show that this preference "$\leq_x$" is weak consistent. In fact, let x and y be two elements of X with $x \leq_x y \leq^u x$. By (ii), we have

$$G(x, x) \leq G(x, y) \leq G(x, x)$$

which yields equality $G(x, x) = G(x, y)$. Consider a net $\{x_\alpha\}_{\alpha \in I}$ by choosing $x_\alpha \in \{x, y\}$ so that for every $\alpha \in I$ there are β and $\gamma \in I$ such that $x_\beta = x$ and $x_\gamma = y$. Then the real net $\{G(x,x_\alpha)\}_{\alpha\in I}$ is constant, hence convergent. By (iii), the net $\{x_\alpha\}_{\alpha\in I}$ is Cauchy, by which $x=y$. This proves that the preference is acyclic and weak consistent as well. Now we prove that for an element $a\in X$, the upper section $S(a)$ is P-complete. To this end let $\{x_\alpha\}_{\alpha\in I}$ be a strictly increasing net in $S(a)$. By (ii), the net $\{G(a,x_\alpha)\}_{\alpha\in I}$ is increasing and bounded above, hence it converges. By (iii) the net $\{x_\alpha\}_{\alpha\in I}$ is Cauchy, and by (i) it converges to some limit in $S(a)$. It is clear that that limit belongs to $S(x_\alpha)$ for all $\alpha\in I$ by the same hypothesis (i). Hence the family $\{X\setminus S(x_\alpha) : \alpha\in I\}$ does not cover $S(a)$ which proves that $S(a)$ is P-complete. It remains to apply Theorem 9 in [31] to complete the proof. □

The conclusion of the lemma is important: it shows not only the existence of a maximal element, but also that, starting from any point a (action, state) some improving path ends in a maximal element (called also a behavioral trap, see Section 4) where agents prefer to stay than to move.

Let us derive a generalized version of a theorem by Brondsted [14, Theorem 1] on existence of maximal elements in uniform spaces. Recall that (X,Γ) where Γ is a family of subsets of the product space $X\times X$ is a uniform space if Γ is a uniformity on X, that is, (i) every element of Γ contains the diagonal $\{(x,x):\ x\in X\}$; (ii) If U and V are elements of Γ, their intersection $U\cap V$ belongs to Γ; (iii) For every $U\in\Gamma$, there is $V\in\Gamma$ such that $(x,y),(y,z)\in V \implies (x,z)\in U$; (iv) If $U\in\Gamma$, then $U^{-1}\in\Gamma$, where $U^{-1}=\{(x,y)\in X\times X:\ (y,x)\in U\}$; and (v) If $U\in\Gamma$, then, any set $V\subset X\times X$ containing U belongs to Γ.

Theorem 4.2 *Assume that (X,Γ) is a uniform Hausdorff space of actions equipped with a worthwhile to change preference "$\leq_x$" generated by a real function Δ on $X\times X$. Suppose further that for every action $a\in X$ the following conditions hold:*

- (i) *The function $x\in X\longmapsto\Delta(a,x)\in I\!R$ is bounded above on X;*
- (ii) *The upper section $S(a)$ is a complete set of (X,Γ);*
- (iii) *The function Δ has decreasing returns: $\Delta(x,z)-\Delta(x,y)\geq k(\Delta(y,z)-\Delta(y,y))$ for all $x,y,z\in X$, $y\leq_y z$, and some k, with $0<k\leq 1$;*
- (iv) *For every $U\in\Gamma$ there is a positive $\delta>0$ such that $x\leq^u y$ and $\Delta(a,y)-\Delta(a,x)<\delta$ imply $(x,y)\in U$.*

Then, starting from $a\in X$, there exists an improving path from a to a maximal action a_* of X.

Proof. We wish to apply Lemma 4.1 to $G(a,\cdot)=\Delta(a,\cdot)$. It suffices to prove that for a given action $a\in X$ the function $\Delta(a,\cdot)$ is increasing with respect to the transitive closure "$\leq^u$" and that condition (iii) of Lemma 4.1 is satisfied. Let $x\leq^u y$, say $x=x_0\leq_{x_0}x_1\cdots\leq_{x_{n-1}}x_n=y$ for some $x_1,\ldots,x_{n-1}\in X$. One has

$$\begin{aligned}\Delta(a,y)-\Delta(a,x) &= \Delta(a,y)-\Delta(a,x_{n-1})+\Delta(a,x_{n-1})-\Delta(a,x_{n-2})+\cdots+\Delta(a,x_1)-\Delta(a,x)\\ &\geq k(\Delta(x_{n-1},x_n)-\Delta(x_{n-1},x_{n-1})+\cdots+\Delta(x_0,x_1)-\Delta(x_0,x_0))\\ &\geq 0\end{aligned}$$

in which the latter inequalities are obtained from (iii) and from the definition of the worthwhile to change preference. Hence $\Delta(a,\cdot)$ is increasing. Furthermore, if $\{x_\alpha\}_{\alpha\in I}$ is an increasing net in $S(a)$ and the net $\{\Delta(a, x_\alpha)\}_{\alpha \in I}$ converges, then for a small positive number δ, one can find $\alpha \in I$ sufficiently large such that

$$\Delta(a, x_\beta) - \Delta(a, x_\gamma) < \delta$$

for all $\beta > \gamma > \alpha$. By (iv), for every element U of the uniformity Γ, there is $\alpha \in I$ such that $(x_\beta, x_\gamma) \in U$ whenever $\beta > \gamma$ are large. This proves that $\{x_\alpha\}_{\alpha \in I}$ is Cauchy. It remains to apply Lemma 4.1 to complete the proof. $\square$

Let us make a short discussion on the assumptions (iii) and (iv) of Theorem 4.2. Assumption (iii) is a weak superlinearity assumption, which means that the marginal payoff $\Delta(x, z) - \Delta(x, y)$, to move from y to z, from the point of view of x, is larger than a portion k of the marginal payoff $\Delta(y, z) - \Delta(y, y)$, to move from y to z, from the point of view of y. In other words, let the new function $\Lambda_x(y, z) = \Delta(x, z) - \Delta(x, y)$ which defines the variable preference $y \leq_x z$ where $x = x_n$ is the current statu quo point. Assumption (iii) yields $\Lambda_x(y, z) \geq k\Lambda_y(y, z), 0 < k \leq 1$ for all $x, y, z \in X$. We obtain $\Lambda_y(y, z) \geq 0 \Longrightarrow \Lambda_x(y, z) \geq 0$ for all $x \in X$. Then, if $x = x_n$, $y = x_{n+1}$, this means that, along an improving path $\{x_n\}$, the agent experiences no regret. If he prefers now z to y from the point of view of his new statu quo position $y = x_{n+1}$, he would have prefered z to y from the old point of view of $x = x_n$, and more generally from any other past point of view.
Assumption (iv) is a satisficing hypothesis (more explanation is given in Section 5). We underline that no compactness assumption is required for the existence of maximal elements in the above theorem which offers a wide range of its application.

We close up this section by presenting a concept of optimizing paths in relation with improving paths. Let $x_0 \in X$ be given. We say a finite collection $\{x_1, ..., x_n\}$ of elements of X is an optimizing path starting from x_0 if for every $i = 1, \ldots, n$ one has $x_{i-1} \leq_{x_{i-1}} x_i$ and x_i is a maximal element with respect to the preference "$\leq_{x_{i-1}}$". The element x_0 is said to be self-maximal if the solitary collection $\{x_0\}$ is an optimizing path starting from x_0.
It is clear that any optimizing path is improving, and that the converse is not always true. Moreover, if an improving path ends at a maximal point x_* whose preference agrees with preferences of its dominant elements in the sense that $x_* \leq_{x_*} y$ and $y \leq_y x_*$ imply $y \leq_{x_*} x_*$, then this maximal element is self-maximal. In particular when an improving path ends at a unique maximal element, it ends at a self-maximal element.

In many economic models improving paths are more economizing than optimizing paths because the latter ones are generally very costly because of the necessity of repeated full exploration of the state space each step, to discover the new preference, using current limited resources which become not available for current exploitation (the current benefits for having optimized the current step.) Moreover, most existing optimizing techniques allow to find local optima only, which may greatly improve the current state, but are not guaranteed to be globally optimal. On the other hand being at a current state, because of lacking knowledge, resources, time, energy and other factors, the agents are unable to explore the whole space to find a maximal element, and so they are happy with exploring neighboring points to improve. Improving step by step with small sacrifices is essential for a worthwhile to change model and in full concord with the idea of "muddling through" processes by Lindblom or satisficing processes by Simon we have mentioned in the introduction.

5 Behavioral traps

Let X be a space of action equipped with a worthwhile to change preference "$\leq_x$" that is defined by the function Δ as given in (3). We assume just for simplicity that $\Delta(x,x) = 0$ for all $x \in X$. It follows from the definition that an action x_* is maximal if for every $y \in X$ either of the following inequalities holds

$$\begin{aligned} \Delta(y, x_*) &\geq 0 && (10) \\ \Delta(x_*, y) &< 0. && (11) \end{aligned}$$

Definition 5.1 *An action x_* is called ideal maximal if* (10) *is true for all $y \in X$, and it is called a behavioral trap if* (11) *is true for all $y \in X, y \neq x_*$.*

An action is ideal if it is worthwhile to change to it from every action of the space; and an action is a behavioral trap if from there it is not worthwhile to change to other actions. It is clear that ideal actions and behavioral traps are maximal, but the converse is not true. The concept of behavioral trap is widely used in psychology.

Consider the case when X is a metric space. The metric $\mu(x,y)$ between two actions represents an index of dissimilarity between them. It can be a minimum number of elementary operations that are necessary to perform y instead of x. Then the cost to change from x to y depends on this distance. For instance, if $e > 0$ is the cost of doing an elementary operation, the cost $C(x,y)$ to change from x to y must be larger than the amount $e\mu(x,y)$. In many situations the disutility function is linear, say $D(t) = \alpha t$ for some $\alpha > 0$, one has $R(x,y) \geq \alpha e \mu(x,y)$. Without loss of generality one may assume that $\alpha e = 1$.

We now derive an existence result for behavioral traps that can be reached by a path of worthwhile changes.

Corollary 5.2 *Assume that (X, μ) is a metric space and that a worthwhile to change relation "$\leq_x$" on X is determined by a function $\Delta(x,y) = M(x,y) - R(x,y)$ with $R(x,y) \geq \mu(x,y)$ for all $x, y \in X$. Suppose further that for each action $a \in X$ the following conditions hold*

(*i*) *The motivation to change function $M(\cdot,\cdot)$ is superlinear with $M(x,x) = 0$ for every $x \in X$ and $M(a,\cdot)$ is bounded above;*

(*ii*) *The resistance to change function $R(\cdot,\cdot)$ satisfies $R(x,y) = 0$ if and only if $x = y$;*

(*iii*) *The upper section $S(a)$ is a complete subspace.*

Then from any initial action $a \in X$ there is an improving path from a to a behavioral trap of X.

Proof. We wish to apply Lemma 4.1 to obtain a maximal element in $S(a)$ and then show that it is a behavioral trap. To this end, set $G(x,y) = M(x,y)$ and prove that conditions (*ii*) and (*iii*) of that lemma are satisfied. For (*ii*) let $x \leq^u y$, that is $x = x_0 \leq_{x_0}$

$x_1 \cdots x_{n-1} \leq_{x_{n-1}} x_n = y$ for some $x_1, \ldots, x_{n-1} \in X$. As M is superlinear, one has

$$\begin{array}{rcl} M(a,y) - M(a,x) & = & M(a,y) - M(a,x_{n-1}) + M(a,x_{n-1}) + \cdots + M(a,x_1) - M(a.x) \\ & \geq & M(x_{n-1},x_n) + M(x_{n-2},x_{n-1}) + \cdots + M(x_0,x_1) \\ & \geq & R(x_{n-1},x_n) + R(x_{n-2},x_{n-1}) + \cdots + R(x_0,x_1) \\ & \geq & \mu(x_{n-1},x_n) + \mu(x_{n-2},x_{n-1}) + \cdots + \mu(x_0,x_1) \\ & \geq & \mu(x,y) \end{array}$$

using the fact that $R(x,y) \geq \mu(x,y) \geq 0$ for all $x,y \in X$, which shows that $M(a,\cdot)$ is increasing with respect to " $\leq^u$ ". The boundedness above of $M(a,\cdot)$ is by hypothesis. To prove (iii) of Lemma 4.1 let $\{x_i\}_{i\in I}$ be an increasing net in $S(a)$ such that $\{M(a,x_i)\}_{i\in I}$ converges to some limit. Then for any $\varepsilon > 0$ there is some index $i_0 \in I$ such that

$$|M(a,x_\alpha) - M(a,x_\beta)| < \varepsilon \text{ for all } \alpha,\beta > i_0.$$

For all $\beta > \alpha > i_0$, we have $x_\alpha = y_0 \leq_{y_0} y_1 \cdots \leq_{y_{n-1}} y_n = x_\beta$ for some $y_1, \ldots, y_{n-1} \in X$. We proceed as above to conclude that

$$\varepsilon > M(a,x_\beta) - M(a,x_\alpha) \geq \mu(x_\alpha,x_\beta),$$

showing that $\{x_\alpha\}_{\alpha\in I}$ is Cauchy. By applying Lemma 4.1 we obtain a maximal element $a^* \in S(a)$. Suppose to the contrary that a^* is not a behavioral trap, that is for some $y \in X$ with $y \neq a^*$ one has $\Delta(a^*,y) \geq 0$. Since a^* is maximal, it follows that $\Delta(y,a^*) \geq 0$ too. Both inequalities yield

$$\begin{array}{rcl} M(a^*,y) & \geq & R(a^*,y) \geq \mu(a^*,y) \\ M(y,a^*) & \geq & R(y,a^*) \geq \mu(y,a^*) \end{array}$$

which implyies

$$0 = M(a^*,a^*) \geq M(a^*,y) + M(y,a^*) \geq 2\mu(a^*,y).$$

By this $a^* = y$, a contradiction. □

An application to personal equilibrium. Koszegi [24] and Koszegi-Rabin [23] have defined a personal equilibrium. They consider an agent equipped with a variable preference $V(y|x) \in I\!R$ which is the utility the agent derives from consuming the bundle of good $y \in X$ (a consumption action), given the reference consumption bundle $x \in X$. Then, $x^* \in X$ is a (strict) personal equilibrium if $V(y|x^*) < V(x^*|x^*)$ for all $y \neq x^*$ (Gul-Pesendorfer [16]). Our result shows how, including motivation and resistance to change functions, a personal equilibrium can emerge in a finite number of steps, following a sequence of worthwhile changes from an initial consumption bundle. This is a model of habits and routines formation (Moreno-Oliveira-Soubeyran [34]).

6 Satisficing change

In this section we analyze, in a new way, the concept of satisficing introduced by Simon [42, 43, 44, 46] and its links with the theory of change (see Soubeyran [47] for a model of "satisficing by rejection", and Attouch-Soubeyran [6] for an initial modelization in term of worthwhile changes). What is striking here is that the Brondsted hypothesis (iv) is a multicriteria satisficing concept relative to the motivation and resistance to change criteria ("exploring enough" to be able to "improve enough"). To our knowledge it is the first time the Brondsted theorem [14, Theorem 1] finds a direct economic application, and even more in term of satisficing. Let us show this point.

Let X be a space of actions and let $G : X \to \mathbb{R}$ be a payoff function which is a measure for a utility, profit, pleasure or life happiness. A given threshold level $\widetilde{G} \in \mathbb{R}$ is satisfied if there is an action $x \in X$ whose payoff $G(x)$ is higher that the level $\widetilde{G}$. Such an element always exists if the level $\widetilde{G}$ is less than the supremum of G on X. In a dynamic setting at a given action x one sets a threshold level $\widetilde{G}(x) > G(x)$ and looks for an action y, called satisficing change, such that $G(y) \geq \widetilde{G}(x) > G(x)$. The amount $\alpha(x) := \widetilde{G}(x) - G(x)$ is called satisficing gap. When a cost function is given, satisficing change is defined by reversing inequalities. Assume now that X is equipped with a worthwhile to change preference relation "$\leq_x$ " which is defined by the function $\Delta(x, y) = M(x, y) - R(x, y)$ as described in Section 2. Assume further that $M(x, y) = u(A(x, y))$ and $R(x, y) = d(C(x, y))$ where $A(\cdot, \cdot)$ is an advantage to change function, $C(\cdot, \cdot)$ is a cost function, u and d are strictly increasing functions taking the null value at 0. It follows from the definition that it is worthwhile to change from x to y if both motivation and resistance to change are satisficing.

Proposition 6.1 *It is worthwhile to change from x to y if and only if there exists a level $\rho \geq 0$ such that the cost to change satisfices that level and the advantage to change $A(x, y)$ satisfices the level $u^{-1}(d(\rho))$, that is $C(x, y) \leq \rho$ and $A(x, y) \geq u^{-1}(d(\rho))$.*

Proof. By definition $x \leq_x y$ if and only if $M(x, y) \geq R(x, y)$. Choosing a positive number λ between $M(x, y)$ and $R(x, y)$ we have $A(x, y) \geq u^{-1}(\lambda)$ and $C(x, y) \leq d^{-1}(\lambda)$. It remains to set $\rho = d^{-1}(\lambda)$ to complete the proof. □

By considering ρ as a variable the function $u^{-1}(d(\rho))$ is represented by a curve starting from the origin of the space $\mathbb{R}_+ \times \mathbb{R}_+$ and divides it into two parts. The upper part is exactly the domain containing all points (C, A) of worthwhile to change couples of actions. The shape of the curve $u^{-1}(d(\rho))$ shows how much advantage to change must increase in response to an increase in the cost in order to remain in the worthwhile to change domain. When X is a metric space the cost function depends on the distance, and so the farer y from x is, the greater the advantage to change must be to make the change worthwhile. This explains the strategy that "exploring enough" (when $\mu(x, y)$ is large) must imply "improving enough" ($A(x, y)$ is large).

When u is a concave function (favoring risk aversion) and d is a convex function (favoring risk seeking), the function $\rho \mapsto u^{-1}(d(\rho))$ is convex (we are assuming d and u strictly increasing). If in addition both of them are sharply kinked at 0, we obtain the famous concept of "loss aversion effect" by Tversky and Kahneman [22, 50] that explains people's tendency to strongly prefers avoiding losses to acquiring gains. Except for the case in which both u and d are linear, the degree of loss aversion increases as the loss increases. For instance when $d(t) = t^2, u(t) = \ln(1 + t)$ one has

$$u^{-1}(d(\rho)) = \exp(\rho^2) - 1.$$

For a couple of actions (x, y) with $C(x, y) = 1$ and $A(x, y) = 2$ it is worthwhile to change from x to y, but tripling the cost and the gain, i.e., $C(x, y) = 3, A(x, y) = 6$, leads to unworthy change.

7 Games with worthwhile to change preference relations

Consider a noncooperative game $\mathcal{G} = (X^i, \text{“} \leq_x^i \text{”})_{i\in I}$ where $I = \{1, \ldots, k\}$ is a finite list of players, X^i is the strategy set and “$\leq_x^i$” is a reference-dependent preference relation of the player i on the set $X = \prod_{i\in I} X^i$. As usually for $i \in I$, the set X^{-i} is the product $\prod_{j\in I, j\neq i} X^j$. If $x \in X$, then x^{-i} is obtained from x by removing the ith component x^i and belongs to X^{-i}, and we write also $x = (x^i, x^{-i})$.

Definition 7.1 *An element x_* of X is called a Nash equilibrium of the game $\mathcal{G}$ if it is $\leq_x^i$-maximal on the set $X^i \times \{x_*^{-i}\}$ for all $i \in I$.*

This is a generalized version of the classical Nash equilibrium when the preferences “ $\leq_x^i$ ” are defined by real payoff functions (see also Luc [29, 30] for the case of vector payoff functions). In fact, if each player i has a payoff function G^i defined on X, then the preference relation $x \leq_x^i y$ holds if and only if $G^i(y) - G^i(x) \geq 0$. Then a strategy x_* of X is a Nash equilibrium if and only if

$$G^i(x_*) = \max_{y^i\in X^i} G^i(y^i, x_*^{-i}),\ i \in I.$$

Inertial games. Definition 7.1 includes also the case of inertial games recently studied by Attouch et al. [4, 5]. In an inertial game there is a cost $C^i[(y^i, x^i)|x]$ of unilateral change from a strategy x^i to a strategy y^i by the player i assuming other players do not move. The function

$$P^i[y|x] := G^i(y) - C^i[(y^i, x^i)|x],$$

called an inertial payoff of the player i, expresses the gain the player i earns at y by passing from x to y, taking into account his cost (resistance) to change. This payoff deletes from each usual normal form game payoff $G^i(y^i, y^{-i})$ the cost $C^i[(y^i, x^i)|x]$ of unilateral change of player i from x^i to y^i, starting from x. Thus, the preference relation $x \leq_x^i y$ holds if and only if $P^i[y|x] - P^i[x|x] \geq 0$. In this game a strategy x_* of X is an inertial Nash equilibrium if and only if

$$P^i[x_*|x_*] = \max_{y^i\in X^i} P^i[(y^i, x_*^{-i})|x_*]\ i \in I.$$

The individual preferences of players induce a global preference, called a Nash preference and defined by

$$x \leq_x^N y \ \text{ if and only if there is some } i \in I \ \text{ such that }, x^{-i} = y^{-i}, x \leq_x^i y.$$

Games with worthwhile to change preferences. In this section we assume that each reference-dependent preference “$\leq_x^i$” is a worthwhile to change preference, that is, $x \leq_x^i y$ if and only if $\Delta^i(x, y) \geq 0$, where $\Delta^i(x, y) = M^i(x, y) - R^i(x, y)$ as presented in Section 2. We define a global worthwhile to change function, called a Nash function, by

$$\Delta^N(x, y) = \sum_{i\in I} \Delta^i((x^i, x^{-i}), (y^i, x^{-i})) = M^N(x, y) - R^N(x, y),$$

where $M^N(x,y) = \sum_{i\in I} M^i((x^i,x^{-i}),(y^i,x^{-i}))$ and $R^N(x,y) = \sum_{i\in I} R^i((x^i,x^{-i}),(y^i,x^{-i}))$. This function represents the sum of all worthwhile to change payoffs when each player changes unilaterally. Our global worthwhile to change function generalizes the well known Nikaido Yosida function of a normal form game $G^i(x^i,x^{-i}), i\in I$,

$$\Omega(x,y) = \sum_{i\in I} \left[G^i(y^i,x^{-i}) - G^i(x^i,x^{-i}) \right].$$

In this simpler case, $M^N(x,y) = \Omega(x,y)$ and $R^N(x,y) = 0$. Nash equilibrium can be expressed by Nash preference and Nash worthwhile function.

Lemma 7.2 *Assume the reference-dependent preference relations "$\leq^i_x$" of the game $\mathcal{G}$ are reflexive. The following assertions hold.*

i) x_* is a Nash equilibrium if and only if it is a maximal element of X with respect to the Nash preference relation "$\leq^N_x$".

Assume further that the relations "$\leq^i_x$" are determined by worthwhile to change functions Δ^i with $\Delta^i(x,x) = 0$ for all $x\in X, i\in I$. Then,

ii) if x_* is a Nash equilibrium then $\Delta^N(x_*,y)\leq 0$ for all $y\in X$, provided that

$$i\in I,\ y^i\in X^i,\ x_* \leq^i_{x_*} (y^i,x_*^{-i}) \leq^i_{(y^i,x_*^{-i})} x_* \Rightarrow \Delta^i(x_*,(y^i,x_*^{-i})) = 0;$$

iii) if $\Delta^N(x_*,y)\leq 0$ for all $y\in X$ then x_* is a Nash equilibrium, provided that

$$i\in I,\ y^i\in X^i,\ \Delta^i(x_*,(y^i,x_*^{-i})) = 0 \Rightarrow \Delta^i((y^i,x_*^{-i}),x_*)\geq 0.$$

In particular, if $\Delta^i, i\in I$ are anti-symmetric at x_* (that is $\Delta(x_*,y) = -\Delta(y,x_*)$), then x_* is a Nash equilibrium if and only if $\Delta^N(x_*,y)\leq 0$ for all $y\in X$.

Proof. The first part of the lemma is clear from the definition. We prove the second part. Assume that x_* is a Nash equilibrium and let $y\in Y$. For every $i\in I$, either $x_* \leq^i_{x_*} (y^i,x_*^{-i})$ is not true, which yields $\Delta^i(x_*,(y^i,x_*^{-i})) < 0$, or $x_* \leq^i_{x_*} (y^i,x_*^{-i})$, which implies $(y^i,x_*^{-i}) \leq^i_{(y^i,x_*^{-i})} x_*$ by hypothesis. By assumption, it follows that $\Delta^i(x_*,(y^i,x_*^{-i})) = 0$ and hence $\Delta^N(x_*,y)\leq 0$.
Conversely, assume that $\Delta^N(x_*,y)\leq 0$ for all $y\in X$. If for some $i\in I$ and $y^i\in X^i$ one has $x_* \leq^i_{x_*} (y^i,x_*^{-i})$, then $\Delta^i(x_*,(y^i,x_*^{-i})) = 0$. The hypothesis of the lemma yields $\Delta^i((y^i,x_*^{-i}),x_*)\geq 0$ by which $(y^i,x_*^{-i}) \leq^i_{(y^i,x_*^{-i})} x_*$. Hence x_* is a Nash equilibrium. □

From now on we consider (X^i,d_i) to be metric spaces which induces the metric $d(x,y) \doteq \sum_{i\in I} d_i(x^i,y^i)$ on the space X. Further, X is equipped with the Nash preference relation "$\leq^N_x$". The upper section $S(a)$ at a strategy $a\in X$ is understood as an upper section with respect to the transitive closure of the Nash preference relation.

The next theorem establishes the existence of a strict (or sharp) "worthwhile to change Nash equilibrium" under very mild assumptions. A usual Nash equilibrium being obtained in the limit, when resistances to change disappear (defining a sequence of worthwhile to change games with strictly positive weights on the resistance to change functions which tend to zero, see Corollary 6.5 in Bianchi-Kassay-Pini [11]). We also obtains the striking result that, starting from an initial profile of actions, players, using, each step, worthwhile to change response functions (optimizing each step being a very special case), can reach a worthwhile to change Nash equilibrium in a finite number of steps. Hence we have a result about a "learning how to play a Nash process", using only worthwhile moves, see Chen-Gazzale [17]. Furthermore the result shows that it is worthwhile to directly unilaterally change from the initial situation to the worthwhile to change Nash equilibrium.

Theorem 7.3 *Assume that the preference relations "$\leq_x^i$" of the game $\mathcal{G}$ are determined by the worthwhile to change functions Δ^i with $M^i(x,x) = 0$ for all $x \in X, i \in I$; $R^i(x,y) = 0$ if and only if $x = y$, and that the space (X,d) is a metric space. Assume further that for every $a \in X$ the following conditions hold.*

- (i) *$S(a)$ is a complete subspace of X;*
- (ii) *The motivation to change functions M^i are superlinear and the functions $x^i \mapsto M^i(a,(x^i,a^{-i}))$ are bounded above on X^i;*
- (iii) *The resistance to change functions R^i are bounded below by the distance: $R^i(x,y) \geq d_i(x^i,y^i)$ for all $x,y \in X$.*

Then, for every strategy $a \in X$ there is an improving path from a to a Nash equilibrium of the game, x_*, such that $\Delta^N(x_*,y) < 0$ for all $y \in X$, $y \neq x_*$. Consequently, for every $i \in I$, $y^i \in X^i$, $y^i \neq x_*^i$, one has $\Delta^i(a,x_*) \geq 0$ and $\Delta^i(x_*,(y^i,x_*^{-i})) < 0$.

Proof. Let us fix a strategy $a \in X$ and consider the complete space $S(a)$ equipped with a new reflexive preference relation $x \leq_x^* y$ if and only if $\Delta^N(x,y) \geq 0$. We wish to apply Corollary 5.2 by setting $M(x,y) = M^N(x,y)$ and $R(x,y) = R^N(x,y)$. Clearly $M(a,\cdot)$ is superlinear; $M(x,x) = 0$ for every $x \in X$, and $R(x,y) = 0$ if and only if $x = y$. Obviously, $M(a,\cdot)$ is bounded above on X because so are the functions on X^i. Thus, according to Corollary 5.2 there is a behavioral trap, x_* in $S(a)$ with respect to the preference relation "$\leq_x^*$". This means that $\Delta^N(x_*,y) < 0$ for all $y \in X$, $y \neq x_*$. From this the conclusions follow. This also proves that x_* is actually a Nash equilibrium. □

Remark 7.4 *Theorem 7.3 shows that,* 1) *starting from $a \in X$, all agents prefer to unilaterally change from a to x_* and* 2) *being at x_*, all agents do not prefer to unilaterally change. This is a "worthwhile to change" Nash equilibrium.*

Remark 7.5 *In Theorem 7.3, assumption (iii) means that resistance to change functions must be higher than distance functions which are polar cases of costs to change: they are high enough for small changes (higher than the square of the distance) and low enough for big changes (lower than the square of the distance); whereas assumption (ii) supposes that motivation to change functions are superlinear. This is the case when motivation to change functions are equal to advantages to change functions $M(x,y) = A^i(x,y) = G^i(y) - G^i(x)$, where $G^i(x)$ is the normal form game payoff of player i. Notice that resistance to change functions are not required to be sublinear. This is a particular case. If, for example desutility of costs to change are linear, $D(C) = \delta C$, $\delta > 0$, resistance to change functions are sublinear, costs to change being sublinear defined by an infimum of costs to change along paths of change (following Soubeyran [49]). As for the existence of maximal elements, no compactness assumption is required. Resistance to change functions do the job.*

The previous theorem is a generalization of the interesting criteria for existence of equilibria via Ekeland's variational principle established in Bianch-Kassay-Pini [11].

Let $X^i, i \in I$ be closed sets of Banach spaces and $f^i : X \times X^i \to \mathbb{R}$. One is interested in finding a point $x_* \in X$ such that

$$f^i(x_*, y^i) \geq 0 \ \ \forall\ i \in I,\ y^i \in X^i.$$

This is a system of equilibrium problems, denoted (SEP). To study (SEP) let us define for each positive ϵ worthwhile to change functions

$$\Delta^i_\epsilon(x, y) = M^i(x, y) - R^i_\epsilon(x, y)$$

with $M^i(x, y) = -f(x, y^i)$ and $R^i_\epsilon(x, y) = \epsilon\|x^i - y^i\|_i$ (here $\|\cdot\|_i$ is a norm in X^i). The game with the worthwhile to change functions Δ^i_ϵ is denoted by $\mathcal{G}_\epsilon$. We obtain Theorem 2.2 of Bianchi-Kassay-Pini [11] in the next corollary.

Corollary 7.6 *Assume that the following conditions hold*

- *(i) for every $x \in X$, the functions $f^i(x, \cdot), i \in I$ are bounded below and lower semi-continuous;*
- *(ii) for every $x \in X$, one has $f^i(x, x^i) = 0, i \in I$;*
- *(iii) for every x, y and z from X on has $f^i(z, x^i) \leq f^i(z, y^i) + f^i(y, x^i), i \in I$.*

Then for every $\epsilon > 0$ and every a in X there is x_* such that for every $i \in I$ and every $x^i \in X^i, x^i \neq x^i_*$ one has

$$f^i(a, x^i_*) + \epsilon\|a^i - x^i_*\|_i \ \leq\ 0 \qquad (12)$$

$$f^i(x_*, x^i) + \epsilon\|x^i_* - x^i\|_i \ >\ 0. \qquad (13)$$

Proof. It is a consequence of the preceding theorem. □

It is now quite standard to obtain a solution to (SEP) when the set X is compact and the functions $f^i(\cdot, y^i)$ are upper semi-continuous for every fixed $y^i \in X^i$ (see Bianchi-Kassay-Pini [11, Proposition 3.2]) by using a sequence of Nash equilibria of the games $\mathcal{G}_{1/n}, n = 1, 2, \ldots$

8 Conclusion

In this paper we have studied a behavioral problem, that is a question on the behavior of bounded rational agents which are not required to optimize each step but make worthwhile to change moves by balancing motivation and resistance to change. Motivation and resistance to change functions cover a lot of situations including emotions, goal setting, goal striving, exploration (editing, evaluation), exploitation, learning, capability building, inertia, habits, routines and norms formation, and learning to play Nash processes. We have established general conditions for existence and reachability of maximal actions in a finite number of worthwhile steps for the worthwhile to change problem. In particular we have obtained conditions of existence of behavioral traps and shown how worthwhile to change processes can be related to satisficing processes. Passing from one isolated agent to interacting agents we have considered a game in which agents are equipped with worthwhile to change variable preferences and proved the existence of Nash equilibrium. It is to underline that no compactness assumption is needed in our approach, resistance to change function doing the job.

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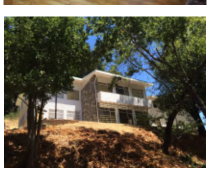

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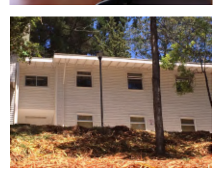

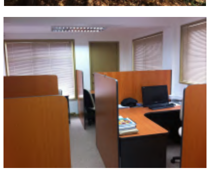

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