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A fully discrete scheme for the pressure-stress formulation of the time-domain fluid-structure interaction problem*

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Abstract

We propose an implicit Newmark method for the time integration of the pressure-stress formulation of a fluid-structure interaction problem. The space Galerkin discretization is based on the Arnold-Falk-Winther mixed finite element method with weak symmetry in the solid and the usual Lagrange finite element method in the acoustic medium. We prove that the resulting fully discrete scheme is well-posed and uniformly stable with respect to the discretization parameters and Poisson ratio, and we provide asymptotic error estimates. Finally, we present numerical tests to confirm the asymptotic error estimates predicted by the theory.

Mathematics Subject Classification. 65N30, 65M12, 65M15, 74H15

Keywords. mixed finite elements, fluid-solid interaction, error estimates

1 Introduction

Recently, the time-domain fluid-structure interaction problem has been formulated in [13] by considering the stress tensor and the fluid pressure as primary variables. The resulting variational problem is symmetric and immune to the locking phenomenon that generally affects displacement based formulations in the nearly incompressible case. Indeed, the convergence analysis presented in [13] revealed that the space semi-discrete Galerkin scheme based on the Arnold-Falk-Winther mixed finite element method with weak symmetry in the solid and the Lagrange finite element method in the acoustic medium is uniformly stable with respect to the space discretization parameter and the Poisson ratio. We also point out that the method provides a direct approximation of the stress tensor, which is the variable of interest in many applications. We refer to [13] for more details and for a comparison with the formulations proposed in [10] and [6].

This paper completes the study given in [13] by carrying out the convergence analysis of an implicit time integration based on the Newmark trapezoidal rule. Following the steps given in [12, Section

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6], we establish the unconditional stability of the resulting fully discrete method when the mesh parameters h and Δt go to 0 and when the Lamé coefficient λ tends to infinity. Finally, we prove that if the $k^{\rm th}-$order Arnold-Falk-Winther element and the $k^{\rm th}-$order Lagrange element ($k \geq 1$) are used in the solid and the fluid domains, respectively, then the error exhibits a combined space-time asymptotic behaviour given by $O(h^k) + O((\Delta t)^2)$.

The rest of the paper is organized as follows. We begin by introducing in Section 2 some basic notations and properties needed in the forthcoming analysis. In Section 3 and 4 we summarize the results obtained in [13] which will be required to present our numerical scheme. Then, in Section 5 we use an implicit Newmark method to obtain a fully discrete version of the problem and carry out its convergence analysis. Finally, in Section 6 we present numerical results that confirm the theoretical convergence estimates.

2 Notations and preliminary results

In what follows, $\boldsymbol{I}$ denotes the identity matrix of $\mathbb{R}^{d\times d}$ ($d = 2, 3$), and $\mathbf{0}$ represents the null vector in $\mathbb{R}^d$ or the null tensor in $\mathbb{R}^{d\times d}$. In addition, given $\boldsymbol{\tau} := (\tau_{ij})$ and $\boldsymbol{\sigma} := (\sigma_{ij}) \in \mathbb{R}^{d\times d}$, we define as usual the transpose tensor $\boldsymbol{\tau}^{\mathtt{t}} := (\tau_{ji})$, the trace $\mathrm{tr}\,\boldsymbol{\tau} := \sum_{i=1}^d \tau_{ii}$, the deviatoric tensor $\boldsymbol{\tau}^{\mathtt{D}} := \boldsymbol{\tau} - \frac{1}{d}\,(\mathrm{tr}\,\boldsymbol{\tau})\,\boldsymbol{I}$, and the tensor inner product $\boldsymbol{\tau} : \boldsymbol{\sigma} := \sum_{i,j=1}^d \tau_{ij}\sigma_{ij}$. We now let Ω be a polyhedral Lipschitz bounded domain of $\mathbb{R}^d$, with boundary $\partial\Omega$, and denote by $\mathcal{D}(\Omega)$ the space of infinitely differentiable functions with compact support in Ω. For $s \in \mathbb{R}$, $\|\cdot\|_{s,\Omega}$ stands indistinctly for the norm of the Hilbertian Sobolev spaces $\mathrm{H}^s(\Omega)$, $\mathrm{H}^s(\Omega)^d$ or $[\mathrm{H}^s(\Omega)]^{d\times d}$, with the convention $\mathrm{H}^0(\Omega) := \mathrm{L}^2(\Omega)$. We also denote by $(\cdot,\cdot)_{0,\Omega}$ the inner product in $\mathrm{L}^2(\Omega)$, $\mathrm{L}^2(\Omega)^d$ or $[\mathrm{L}^2(\Omega)]^{d\times d}$. We notice that the orthogonal decomposition $[\mathrm{L}^2(\Omega)]^{d\times d} = [\mathrm{L}^2(\Omega)]^{d\times d}_{\rm sym} \oplus [\mathrm{L}^2(\Omega)]^{d\times d}_{\rm skew}$ holds true with

$$[\mathrm{L}^2(\Omega)]^{d\times d}_{\rm sym} := \left\{\boldsymbol{\tau} \in [\mathrm{L}^2(\Omega)]^{d\times d}; \quad \boldsymbol{\tau} = \boldsymbol{\tau}^{\mathtt{t}}\right\} \quad \text{and} \quad [\mathrm{L}^2(\Omega)]^{d\times d}_{\rm skew} := \left\{\boldsymbol{\tau} \in [\mathrm{L}^2(\Omega)]^{d\times d}; \quad \boldsymbol{\tau} = -\boldsymbol{\tau}^{\mathtt{t}}\right\}.$$

We introduce the Hilbert space

$$\mathbf{H}(\mathbf{div},\Omega) := \left\{\boldsymbol{\tau} \in [\mathrm{L}^2(\Omega)]^{d\times d}; \quad \mathbf{div}\,\boldsymbol{\tau} \in \mathrm{L}^2(\Omega)^d\right\},$$

whose norm is given by $\|\boldsymbol{\tau}\|^2_{\mathrm{H}(\mathbf{div},\Omega)} := \|\boldsymbol{\tau}\|^2_{0,\Omega} + \|\mathbf{div}\,\boldsymbol{\tau}\|^2_{0,\Omega}$. In turn, given $p \in [1,+\infty]$, $T > 0$ and a separable Hilbert space V with norm $\|\cdot\|_V$, we let $\mathrm{L}^p((0,T);V)$ be the space of classes of functions $f : (0,T) \to V$ that are Bochner-measurable and such that $\|f\|_{\mathrm{L}^p((0,T);V)} < \infty$, with

$$\|f\|^p_{\mathrm{L}^p((0,T);V)} := \int_0^T \|f(t)\|^p_V \,\mathrm{d}t \quad (1 \leq p < \infty) \quad \text{and} \quad \|f\|_{\mathrm{L}^\infty((0,T);V)} := \operatorname*{ess\,sup}_{[0,T]} \|f(t)\|_V .$$

For any $k \in \mathbb{N}$, we consider the space $\mathcal{C}^k((0,T);V)$ of all functions f with (strong) derivatives $f^{(j)}$ in $\mathcal{C}^0((0,T);V)$ for all $1 \leq j \leq k$, where $\mathcal{C}^0((0,T);V)$ stands for the Banach space consisting of all continuous functions $f : [0,T] \to V$. We will use also denote $\dot{f}$ and $\ddot{f}$ the first and second derivatives with respect to the variable t. Furthermore, we will use the Sobolev space

$$\mathrm{W}^{1,p}((0,T);V) := \Big\{f : \ \exists g \in \mathrm{L}^p((0,T);V) \text{ and } \exists f_0 \in V \text{ such that}$$
$$f(t) = f_0 + \int_0^t g(s)\,\mathrm{d}s \quad \forall\, t \in [0,T]\Big\}.$$

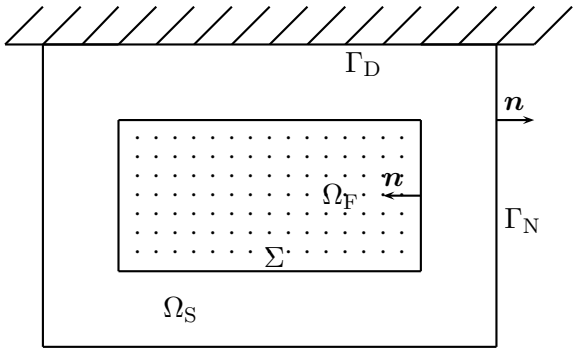

Figure 3.1: Fluid and solid domains

With the convention that $\mathrm{W}^{0,p}((0,T);V) = \mathrm{L}^p((0,T);V)$, the space $\mathrm{W}^{k,p}((0,T);V)$ is defined recursively for all $k \in \mathbb{N}$, that is

$$\mathrm{W}^{k,p}((0,T);V) := \Bigg\{ f : \ \exists g \in \mathrm{W}^{k-1,p}((0,T);V) \text{ and } \exists f_0 \in V \\ \text{such that} \quad f(t) = f_0 + \int_0^t g(s)\,\mathrm{d}s \quad \forall\, t \in [0,T] \Bigg\}.$$

Throughout this paper we use C (with or without subscripts) to denote generic constants independent of the parameters indicated at each instance. We point out that these constants may take different values at different places.

3 Stress-pressure variational formulation of the model problem

We consider a solid body represented by a connected polyhedral Lipschitz domain $\Omega_{\rm S}$ whose boundary is given by two connected components Σ and Γ. The cavity $\Omega_{\rm F}$ delimited by the inner boundary Σ is filled with an homogeneous, inviscid and compressible fluid (see Figure 3.1). Our objective is to compute the linear oscillations that take place in the fluid-solid domain $\Omega := \Omega_{\rm S} \cup \Sigma \cup \Omega_{\rm F}$, under the action of a given loading $\boldsymbol{f} : (0,T] \times \Omega_{\rm S} \to \mathbb{R}^n$ prescribed in the solid domain. We assume that the solid is fixed at a nonempty part $\Gamma_{\rm D}$ of the external boundary $\Gamma := \partial\Omega$ and impose a traction-free boundary condition on its complement $\Gamma_{\rm N} := \Gamma \setminus \Gamma_{\rm D}$. We denote $\boldsymbol{n}$ the outward unit normal vector to $\Gamma \cup \Sigma$ and select on Σ the orientation that points outward to $\Omega_{\rm F}$. More precisely, the mathematical model associated to the physical phenomenon under interest is given by the set of equations

$$\rho_{\rm S}\ddot{\boldsymbol{u}} - \mathbf{div}\,\mathcal{C}\boldsymbol{\varepsilon}(\boldsymbol{u}) = \boldsymbol{f} \qquad \text{in } \Omega_{\rm S} \times (0,T], \tag{3.1}$$

$$c^{-2}\ddot{p} - \Delta p = 0 \qquad \text{in } \Omega_{\rm F} \times (0,T], \tag{3.2}$$

$$\mathcal{C}\boldsymbol{\varepsilon}(\boldsymbol{u})\boldsymbol{n} + p\boldsymbol{n} = \boldsymbol{0} \qquad \text{on } \Sigma \times (0,T], \tag{3.3}$$

$$\frac{\partial p}{\partial \boldsymbol{n}} + \rho_{\rm F}\ddot{\boldsymbol{u}} \cdot \boldsymbol{n} = 0 \qquad \text{on } \Sigma \times (0,T], \tag{3.4}$$

$$\boldsymbol{u} = \boldsymbol{0} \qquad \text{on } \Gamma_{\rm D} \times (0,T], \tag{3.5}$$

$$\mathcal{C}\boldsymbol{\varepsilon}(\boldsymbol{u})\boldsymbol{n} = \boldsymbol{0} \qquad \text{on } \Gamma_{\rm N} \times (0,T], \tag{3.6}$$

with the corresponding initial conditions. Here, p is the fluid pressure, $\mathcal{C}:\ \mathbb{R}^{d\times d}\to\mathbb{R}^{d\times d}$ is the Hooke operator given by

$$\mathcal{C}\boldsymbol{\tau} := \lambda\,(\mathrm{tr}\,\boldsymbol{\tau})\,\boldsymbol{I} + 2\mu\boldsymbol{\tau} \qquad \forall\,\boldsymbol{\tau}\in\mathbb{R}^{d\times d}\,,$$

$\boldsymbol{\varepsilon}(\boldsymbol{u})$ is the linearized strain tensor, which is given, in terms of the solid displacement field $\boldsymbol{u}$, by

$$\boldsymbol{\varepsilon}(\boldsymbol{u}) := \frac{1}{2}\Big\{\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^{\mathtt{t}}\Big\}\,,$$

$\rho_{\mathrm{S}} > 0$ is the density of the solid, $\lambda > 0$ and $\mu > 0$ are its Lamé coefficients, $c > 0$ is the acoustic speed in the fluid, and $\rho_{\mathrm{F}} > 0$ is its density.

The stress tensor $\boldsymbol{\sigma} := \mathcal{C}\,\boldsymbol{\varepsilon}(\boldsymbol{u})$, which is imposed here as a primary unknown in the solid, is sought in the Sobolev space

$$\boldsymbol{\mathcal{W}} := \Big\{\boldsymbol{\tau}\in\mathbf{H}(\mathbf{div},\Omega_{\mathrm{S}});\quad \boldsymbol{\tau}\boldsymbol{n} = \mathbf{0}\quad \text{on } \Gamma_{\mathrm{N}}\Big\},$$

while the pressure p belongs to $\mathrm{H}^1(\Omega_{\mathrm{F}})$. These two variables are linked through equation (3.3), which can be interpreted as an implicitly prescribed normal stress on the contact boundary Σ. As we are dealing with a dual formulation in Ω_{S}, this transmission condition becomes essential, and hence we could impose it weakly through a suitable Lagrange multiplier (as we did in [14]), or alternatively, we could incorporate it into the continuous space. Here, we follow [15] and choose the second option by defining the global space

$$\mathbb{X} := \Big\{(\boldsymbol{\tau},q)\in\boldsymbol{\mathcal{W}}\times\mathrm{H}^1(\Omega_{\mathrm{F}});\quad \boldsymbol{\tau}\boldsymbol{n} + q\boldsymbol{n} = \mathbf{0}\qquad \text{on } \Sigma\Big\},$$

which is endowed with the Hilbertian norm $\|(\boldsymbol{\tau},q)\|^2 := \|\boldsymbol{\tau}\|^2_{\mathrm{H}(\mathbf{div},\Omega_{\mathrm{S}})} + \|q\|^2_{1,\Omega_{\mathrm{F}}}$.

We still have to impose a further restriction in $\mathbb{X}$. Indeed, it is essential to take into account the conservation of the angular momentum, which is characterized by the symmetry of the stress tensor. This induces us to consider the closed subspace

$$\mathbb{X}^{\mathrm{sym}} := \big\{(\boldsymbol{\tau},q)\in\mathbb{X};\quad \boldsymbol{\tau} = \boldsymbol{\tau}^{\mathtt{t}}\big\}.$$

We point out that, stable mixed finite elements for the linear elastostatic problem have been arduous to construct because of this symmetry restriction (cf. [1, 2, 3, 5, 7, 9]). One of the prevailing techniques [1, 3, 7, 9] used to deal with this difficulty consists in imposing weakly the symmetry through the introduction of a Lagrange multiplier, which turns out to be equal to the rotation $\boldsymbol{r} := \frac{1}{2}\big\{\nabla\boldsymbol{u} - (\nabla\boldsymbol{u})^{\mathtt{t}}\big\}$. Recently, this mixed finite element strategy with reduced symmetry has been successfully applied to the elasticity eigenproblem [17], to the indefinite elasticity problem [16], to elastodynamics [4, 12], and to time-domain fluid-structure interaction problems [13]. It is important to bear in mind that, in what follows, there will be an underlying Lagrange multiplier (corresponding to the symmetry restriction) that we have chosen to hide for economy in notations. We refer to [12] (or its preliminary summarized version [11]) for a similar analysis for the elastodynamics in which the rotation variable is maintained as an active unknown.

We now notice that $\mathbb{X}^{\mathrm{sym}}$ is dense in the space

$$\mathbb{H}^{\mathrm{sym}} := [\mathrm{L}^2(\Omega_{\mathrm{S}})]^{d\times d}_{\mathrm{sym}}\times\mathrm{L}^2(\Omega_{\mathrm{F}})$$

endowed with the norm $\|(\boldsymbol{\tau},q)\|^2_0 := \|\boldsymbol{\tau}\|^2_{0,\Omega_{\mathrm{S}}} + \|q\|^2_{0,\Omega_{\mathrm{F}}}$. This allows us to pose the stress-pressure variational formulation of the fluid-solid interaction problem in the following terms (see [13, eq.(3.11)]

for details):

$$\begin{aligned}&\text{Find } (\boldsymbol{\sigma},p)\in \mathrm{L}^\infty((0,T);\mathbb{X}^{\mathrm{sym}})\cap \mathrm{W}^{1,\infty}((0,T);\mathbb{H}^{\mathrm{sym}}) \text{ such that}\\ &\Big((\ddot{\boldsymbol{\sigma}},\ddot{p})(t),(\boldsymbol{\tau},q)\Big)_{\mathcal{C}}+A\Big((\boldsymbol{\sigma},p)(t),(\boldsymbol{\tau},q)\Big)=-\rho_{\mathrm{S}}^{-1}\big(\boldsymbol{f}(t),\mathbf{div}\,\boldsymbol{\tau}\big)_{0,\Omega_{\mathrm{S}}}\quad \forall(\boldsymbol{\tau},q)\in\mathbb{X}^{\mathrm{sym}},\\ &\big(\boldsymbol{\sigma}(0),p(0)\big)=(\boldsymbol{\sigma}_0,p_0),\qquad \big(\dot{\boldsymbol{\sigma}}(0),\dot{p}(0)\big)=(\boldsymbol{\sigma}_1,p_1),\end{aligned}\tag{3.7}$$

where

$$\Big((\boldsymbol{\sigma},p),(\boldsymbol{\tau},q)\Big)_{\mathcal{C}}:=(\mathcal{C}^{-1}\boldsymbol{\sigma},\boldsymbol{\tau})_{0,\Omega_{\mathrm{S}}}+\frac{1}{\rho_{\mathrm{F}}c^2}(p,q)_{0,\Omega_{\mathrm{F}}}$$

and

$$A\Big((\boldsymbol{\sigma},p),(\boldsymbol{\tau},q)\Big):=\rho_{\mathrm{S}}^{-1}(\mathbf{div}\,\boldsymbol{\sigma},\mathbf{div}\,\boldsymbol{\tau})_{0,\Omega_{\mathrm{S}}}+\rho_{\mathrm{F}}^{-1}(\nabla p,\nabla q)_{0,\Omega_{\mathrm{F}}}.$$

Here, $\boldsymbol{f}\in \mathrm{L}^1((0,T);\mathrm{L}^2(\Omega_{\mathrm{S}})^d)$ is a given body force in Ω_{S} and $(\boldsymbol{\sigma}_0,p_0)\in\mathbb{X}^{\mathrm{sym}}$ and $(\boldsymbol{\sigma}_1,p_1)\in\mathbb{H}^{\mathrm{sym}}$ are prescribed initial data.

The stability of our analysis with respect to λ when this parameter tends to infinity relays essentially on the following result.

Lemma 3.1. *There exist constants $c_2\geq c_1>0$ independent of λ such that*

$$c_1\,\|(\boldsymbol{\tau},q)\|^2\leq\|(\boldsymbol{\tau},q)\|_{0,\mathcal{C}}^2+A\Big((\boldsymbol{\tau},q),(\boldsymbol{\tau},q)\Big)\leq c_2\,\|(\boldsymbol{\tau},q)\|^2\quad\forall(\boldsymbol{\tau},q)\in\mathbb{X},\tag{3.8}$$

where $\|(\boldsymbol{\tau},q)\|_{0,\mathcal{C}}^2:=\Big((\boldsymbol{\tau},q),(\boldsymbol{\tau},q)\Big)_{\mathcal{C}}$.

Proof. The bound from above follows immediately from the fact that

$$(\mathcal{C}^{-1}\boldsymbol{\sigma},\boldsymbol{\tau})_{0,\Omega_{\mathrm{S}}}\;=\;\frac{1}{2\mu}\int_{\Omega_{\mathrm{S}}}\boldsymbol{\sigma}^{\mathtt{D}}:\boldsymbol{\tau}^{\mathtt{D}}+\frac{1}{d(2\mu+d\lambda)}\int_{\Omega_{\mathrm{S}}}(\mathrm{tr}\,\boldsymbol{\sigma})(\mathrm{tr}\,\boldsymbol{\tau})$$

is bounded by a constant independent of λ. The left inequality may be found in [17, Lemma 2.1]. □

The well-posedness of problem (3.7) is established as follows (cf. [13, Theorem 3.1]) .

Theorem 3.1. *Assume that $\boldsymbol{f}\in\mathrm{W}^{1,1}((0,T);\mathrm{L}^2(\Omega_{\mathrm{S}})^d)$. Then, problem* (3.7) *admits a unique solution $(\boldsymbol{\sigma},p)\in\mathcal{C}^0((0,T);\mathbb{X}^{sym})\cap\mathcal{C}^1((0,T);\mathbb{H}^{sym})$. Moreover, there exists a constant $C>0$, independent of λ and T, such that*

$$\operatorname*{ess\,sup}_{[0,T]}\|(\boldsymbol{\sigma},p)(t)\|+\operatorname*{ess\,sup}_{[0,T]}\|(\dot{\boldsymbol{\sigma}},\dot{p})(t)\|_{0,\mathcal{C}}\leq\;C\,T\,\Big\{\,\|\boldsymbol{f}\|_{\mathrm{W}^{1,1}(\boldsymbol{L}^2(\Omega_{\mathrm{S}}))}+\|(\boldsymbol{\sigma}_0,p_0)\|+\|(\boldsymbol{\sigma}_1,p_1)\|_0\,\Big\}.$$

Although problem (3.7) is well-posed in the sense of Hadamard, it turns out that a compatibility condition must be imposed to the initial data $(\boldsymbol{\sigma}_0,p_0)\in\mathbb{X}^{\mathrm{sym}}$ and $(\boldsymbol{\sigma}_1,p_1)\in\mathbb{H}^{\mathrm{sym}}$ in order to remove non physical components from the solution. These spurious modes are due to the fact that the seminorm $A\Big((\boldsymbol{\tau},q),(\boldsymbol{\tau},q)\Big)^{1/2}$ admits the nontrivial kernel $\mathbb{K}$ in $\mathbb{X}_c^{\mathrm{sym}}$ given by

$$\mathbb{K}:=\Big\{(\boldsymbol{\tau},q)\in\mathbb{X}_c^{\mathrm{sym}};\quad\mathbf{div}\,\boldsymbol{\tau}=0\Big\},$$

where $\mathbb{X}_c^{\mathrm{sym}}:=\mathbb{X}_c\cap\mathbb{X}^{\mathrm{sym}}$, with $\mathbb{X}_c:=\{(\boldsymbol{\tau},q)\in\mathbb{X};\quad q=\text{constant}\}$. It has been shown in [17] that all physically relevant eigenfunctions of the eigenproblem associated with the linear evolution problem (3.7) lie on the orthogonal $\mathbb{K}^\perp$ to $\mathbb{K}$ in $\mathbb{X}^{\mathrm{sym}}$ with respect to the inner product $\big(\cdot,\cdot\big)_{\mathcal{C}}$, i.e.,

$$\mathbb{K}^\perp:=\Big\{(\boldsymbol{\sigma},p)\in\mathbb{X}^{\mathrm{sym}};\quad\Big((\boldsymbol{\sigma},p),(\boldsymbol{\tau},q)\Big)_{\mathcal{C}}=0\quad\forall(\boldsymbol{\tau},q)\in\mathbb{K}\Big\},$$

and $\mathbb{K}$ is the eigenspace associated with an infinite-multiplicity eigenvalue equal to 1. A physically meaningful solution of problem (3.7) should then belong to $\mathbb{K}^\perp$ for all $t \in [0,T]$. This property is simply achieved by imposing the condition at initial time, i.e.,

$$(\boldsymbol{\sigma}_0, p_0) \in \mathbb{K}^\perp \qquad \text{and} \qquad (\boldsymbol{\sigma}_1, p_1) \in \mathbb{K}^\perp.$$

It is shown in [13, Theorem 2.1] that there exits a linear and bounded operator

$$\begin{array}{rcl} \mathbf{D} : \mathbb{K}^\perp & \to & \mathrm{L}^2(\Omega_{\mathrm{S}})^d \times [\mathrm{L}^2(\Omega_{\mathrm{S}})]^{d\times d}_{\mathrm{skew}} \\ (\boldsymbol{\sigma}, p) & \mapsto & (\boldsymbol{u}, \boldsymbol{r}) := \mathbf{D}(\boldsymbol{\sigma}, p) \end{array}$$

uniquely characterized, for any $(\boldsymbol{\sigma}, p) \in \mathbb{K}^\perp$, by the unique solution $(\boldsymbol{u}, \boldsymbol{r}) \in \mathrm{L}^2(\Omega_{\mathrm{S}})^d \times [\mathrm{L}^2(\Omega_{\mathrm{S}})]^{d\times d}_{\mathrm{skew}}$ of

$$(\boldsymbol{r}, \boldsymbol{\tau})_{0,\Omega_{\mathrm{S}}} + (\boldsymbol{u}, \mathbf{div}\, \boldsymbol{\tau})_{0,\Omega_{\mathrm{S}}} = -\Big((\boldsymbol{\sigma}, p), (\boldsymbol{\tau}, \xi)\Big)_{\mathcal{C}} \quad \forall (\boldsymbol{\tau}, \xi) \in \mathbb{X}_c.$$

Moreover, if $(\boldsymbol{u}, \boldsymbol{r}) := \mathbf{D}(\boldsymbol{\sigma}, p)$, then it can be shown that $\boldsymbol{u}$ is none other than the displacement field, with $\boldsymbol{u}(t) \in [\mathrm{H}^1(\Omega_{\mathrm{S}})]^d \ \forall\, t > 0$, and $\boldsymbol{r} = \frac{1}{2}\big\{\nabla \boldsymbol{u} - (\nabla \boldsymbol{u})^{\mathtt{t}}\big\}$ is the rotation. The following result (cf. [13, Theorem 3.2]) establishes the relation between the solution $(\boldsymbol{\sigma}, p)$ of problem (3.7) and the weak solution of the displacement-pressure formulation of the fluid-structure interaction problem.

Theorem 3.2. *Assume that the initial data of problem* (3.7) *are such that* $(\boldsymbol{\sigma}_0, p_0)$, $(\boldsymbol{\sigma}_1, p_1) \in \mathbb{K}^\perp$, *and let* $(\boldsymbol{u}_0, \boldsymbol{r}_0) := \mathbf{D}(\boldsymbol{\sigma}_0, p_0)$ *and* $(\boldsymbol{u}_1, \boldsymbol{r}_1) := \mathbf{D}(\boldsymbol{\sigma}_1, p_1)$. *If* $(\boldsymbol{\sigma}, p)$ *is the solution of* (3.7), *then the pair* $(\boldsymbol{u}, p)$, *with*

$$\boldsymbol{u}(t) := \int_0^t \Big\{ \int_0^s \rho_{\mathrm{S}}^{-1}\Big(\mathbf{div}\, \boldsymbol{\sigma}(z) + \boldsymbol{f}(z)\Big)\, dz \Big\}\, ds + \boldsymbol{u}_0 + t\boldsymbol{u}_1,$$

solves the displacement-pressure formulation of the fluid-structure interaction problem given by the equations (3.1)-(3.6) *subject to the initial conditions* $(\boldsymbol{u}(0), p(0)) = (\boldsymbol{u}_0, p_0)$ *and* $(\dot{\boldsymbol{u}}(0), \dot{p}(0)) = (\boldsymbol{u}_1, p_1)$.

4 Finite element discretization spaces and technical tools

We consider shape regular affine meshes $\mathcal{T}_h$ that subdivide the domain $\bar{\Omega} = \bar{\Omega}_{\mathrm{S}} \cup \bar{\Omega}_{\mathrm{F}}$ into triangles/tetrahedra K of diameter h_K. The parameter $h := \max_{K\in\mathcal{T}_h}\{h_K\}$ represents the mesh size of $\mathcal{T}_h$. In what follows, we assume that each triangle/tetrahedron of $\mathcal{T}_h$ is contained either in $\bar{\Omega}_{\mathrm{S}}$ or in $\bar{\Omega}_{\mathrm{F}}$, and denote

$$\mathcal{T}_h^{\mathrm{S}} := \big\{K \in \mathcal{T}_h; \quad K \subset \bar{\Omega}_{\mathrm{S}}\big\} \quad \text{and} \quad \mathcal{T}_h^{\mathrm{F}} := \big\{K \in \mathcal{T}_h; \quad K \subset \bar{\Omega}_{\mathrm{F}}\big\}.$$

Moreover, we let Σ_h be the triangulation induced by $\mathcal{T}_h$ on Σ, whose elements (edges or triangles) are denoted by T. Next, given an integer $m \geq 0$ and a domain $D \subseteq \mathbb{R}^d$, $\mathcal{P}_m(D)$ denotes the space of polynomials of degree at most m on D. The space of piecewise polynomial functions of degree at most m associated with $\mathcal{T}_h^*$, $* \in \{\mathrm{S}, \mathrm{F}\}$, is denoted by

$$\mathcal{P}_m(\mathcal{T}_h^*) := \big\{v \in L^2(\Omega_*); \quad v|_K \in \mathcal{P}_m(K), \quad \forall K \in \mathcal{T}_h^*\big\}.$$

Similarly, $\mathcal{P}_m(\Sigma_h) := \big\{\phi \in L^2(\Sigma); \quad \phi|_T \in \mathcal{P}_m(T), \quad \forall T \in \Sigma_h\big\}$. In addition, for $k \geq 1$, the finite element spaces

$$\boldsymbol{\mathcal{W}}_h := \mathcal{P}_k(\mathcal{T}_h^{\mathrm{S}})^{d\times d} \cap \boldsymbol{\mathcal{W}}, \quad \boldsymbol{\mathcal{Q}}_h := \mathcal{P}_{k-1}(\mathcal{T}_h^{\mathrm{S}})^{d\times d} \cap [\mathrm{L}^2(\Omega_{\mathrm{S}})]^{d\times d}_{\mathrm{skew}}, \quad \text{and} \quad \mathcal{U}_h := \mathcal{P}_{k-1}(\mathcal{T}_h^{\mathrm{S}})^d,$$

correspond to the k^{th}-order element of the Arnold-Falk-Winther (AFW) family introduced for the mixed formulation of elastostatic problem with reduced symmetry. It is important to notice that

$\boldsymbol{\mathcal{W}}_h^{\rm sym} := \Big\{ \boldsymbol{\tau}_h \in \boldsymbol{\mathcal{W}}_h; \quad \int_{\Omega_{\rm S}} \boldsymbol{\tau}_h : \boldsymbol{s} = 0 \quad \forall \boldsymbol{s} \in \boldsymbol{\mathcal{Q}}_h \Big\}$, which is the weakly symmetric version of $\boldsymbol{\mathcal{W}}_h$, is not a subspace of the symmetric tensors of $\boldsymbol{\mathcal{W}}$. The pressure is approximated in the usual Lagrange finite element space $V_h := \mathcal{P}_k(\mathcal{T}_h^{\rm F}) \cap {\rm H}^1(\Omega_{\rm F})$.

Next, we recall some well-known approximation properties of the finite element spaces introduced above. Given $s > 0$, it is well-known that the usual $k^{\rm th}$-order Brezzi-Douglas-Marini (BDM) interpolation operator (see [8]) $\boldsymbol{\Pi}_h : [{\rm H}^s(\Omega_{\rm S})]^{d\times d} \cap \boldsymbol{\mathcal{W}} \to \boldsymbol{\mathcal{W}}_h$ satisfies for $0 < s \leq 1/2$ the error estimate

$$\|\boldsymbol{\tau} - \boldsymbol{\Pi}_h \boldsymbol{\tau}\|_{0,\Omega_{\rm S}} \leq C h^s \Big\{ \|\boldsymbol{\tau}\|_{s,\Omega_{\rm S}} + \|\mathbf{div}\, \boldsymbol{\tau}\|_{0,\Omega_{\rm S}} \Big\} \qquad \forall \boldsymbol{\tau} \in [{\rm H}^s(\Omega_{\rm S})]^{d\times d} \cap \boldsymbol{\mathcal{W}}.$$

For more regular functions $\boldsymbol{\tau} \in [{\rm H}^s(\Omega_{\rm S})]^{d\times d}$ with $s > 1/2$, it holds

$$\|\boldsymbol{\tau} - \boldsymbol{\Pi}_h \boldsymbol{\tau}\|_{0,\Omega_{\rm S}} \leq C h^{\min\{s,k+1\}} \|\boldsymbol{\tau}\|_{s,\Omega_{\rm S}}, \qquad \forall \boldsymbol{\tau} \in [{\rm H}^s(\Omega_{\rm S})]^{d\times d}. \tag{4.1}$$

Moreover, we have the commuting diagram properties

$$\mathbf{div}(\boldsymbol{\Pi}_h \boldsymbol{\tau}) = U_h(\mathbf{div}\, \boldsymbol{\tau}) \quad \text{and} \quad (\boldsymbol{\Pi}_h \boldsymbol{\sigma})\boldsymbol{n} = \boldsymbol{\pi}_h(\boldsymbol{\sigma}\boldsymbol{n}) \tag{4.2}$$

for all $\boldsymbol{\tau} \in {\rm H}^s(\Omega_{\rm S})^{d\times d} \cap \mathbf{H}(\mathbf{div}, \Omega_{\rm S})$, $s > 0$, where $U_h : {\rm L}^2(\Omega_{\rm S})^d \to \mathcal{U}_h$ is the ${\rm L}^2(\Omega_{\rm S})^d$-orthogonal projector and $\boldsymbol{\pi}_h$ is the vectorial version of π_h, which is the ${\rm L}^2(\Sigma)$-orthogonal projector onto $\mathcal{P}_k(\Sigma_h)$. In addition, we denote by $\boldsymbol{R}_h : [{\rm L}^2(\Omega_{\rm S})]^{d\times d}_{\rm skew} \to \boldsymbol{\mathcal{Q}}_h$ the orthogonal projector with respect to the $[{\rm L}^2(\Omega_{\rm S})]^{d\times d}$-norm, and let $\Pi_h : {\rm H}^1(\Omega_{\rm F}) \to V_h$ be the operator that, given $p \in {\rm H}^1(\Omega_{\rm F})$, is uniquely characterized by

$$(\nabla \Pi_h p, \nabla q)_{0,\Omega_{\rm F}} = (\nabla p, \nabla q)_{0,\Omega_{\rm F}} \quad \forall q \in V_h \quad \text{and} \quad \int_{\Omega_{\rm F}} \Pi_h p = 0\,.$$

Then, there hold

$$\|\boldsymbol{r} - \boldsymbol{R}_h \boldsymbol{r}\|_{0,\Omega_{\rm S}} \leq C h^{\min\{s,k\}} \|\boldsymbol{r}\|_{s,\Omega_{\rm S}} \qquad \forall \boldsymbol{r} \in [{\rm H}^s(\Omega_{\rm S})]^{d\times d} \cap [{\rm L}^2(\Omega_{\rm S})]^{d\times d}_{\rm skew}, \tag{4.3}$$

$$\|\boldsymbol{v} - U_h \boldsymbol{v}\|_{0,\Omega_{\rm S}} \leq C h^{\min\{s,k\}} \|\boldsymbol{v}\|_{s,\Omega_{\rm S}} \qquad \forall \boldsymbol{v} \in {\rm H}^s(\Omega_{\rm S})^d, \tag{4.4}$$

$$|p - \Pi_h p|_{1,\Omega_{\rm F}} \leq C h^{\min\{s,k\}} \|p\|_{1+s,\Omega_{\rm F}} \qquad \forall p \in {\rm H}^{1+s}(\Omega_{\rm F}), \tag{4.5}$$

$$\|\boldsymbol{\varphi} - \boldsymbol{\pi}_h \boldsymbol{\varphi}\|_{0,\Sigma} \leq C h^{\min\{s,k+1\}} \Big(\sum_{T\in\Sigma_h} \|\boldsymbol{\varphi}\|^2_{s,T} \Big)^{1/2} \qquad \forall \boldsymbol{\varphi} \in \prod_{T\in\Sigma_h} {\rm H}^s(T)^d\,. \tag{4.6}$$

We now introduce the discrete energy space $\mathbb{X}_h := \{(\boldsymbol{\tau}, q) \in \boldsymbol{\mathcal{W}}_h \times V_h; \quad \boldsymbol{\tau}\boldsymbol{n} + q\boldsymbol{n} = \mathbf{0} \text{ on } \Sigma\}$, and its subspace $\mathbb{X}_{h,c} = \{(\boldsymbol{\tau}, q) \in \mathbb{X}_h; \quad q = \text{constant}\}$. We also consider their weakly symmetric versions $\mathbb{X}_h^{\rm sym} := \big\{(\boldsymbol{\tau}, q) \in \boldsymbol{\mathcal{W}}_h^{\rm sym} \times V_h; \quad \boldsymbol{\tau}\boldsymbol{n} + q\boldsymbol{n} = \mathbf{0} \text{ on } \Sigma\big\}$ and $\mathbb{X}_{h,c}^{\rm sym} := \mathbb{X}_{h,c} \cap \mathbb{X}_h^{\rm sym}$, respectively. The kernel $\mathbb{K}_h$ of the bilinear form A in $\mathbb{X}_h^{\rm sym}$ is given by

$$\mathbb{K}_h := \Big\{ (\boldsymbol{\tau}, q) \in \mathbb{X}_{h,c}^{\rm sym}; \quad \mathbf{div}\, \boldsymbol{\tau} = \mathbf{0} \Big\}\,,$$

and we notice that, in general, neither $\mathbb{K}_h \subseteq \mathbb{K}$ nor $\mathbb{K}_h^\perp \subseteq \mathbb{K}^\perp$, with

$$\mathbb{K}_h^\perp := \Big\{ (\boldsymbol{\sigma}_h, p_h) \in \mathbb{X}_h^{\rm sym}; \quad \Big((\boldsymbol{\sigma}_h, p_h), (\boldsymbol{\tau}, \xi) \Big)_{\mathcal{C}} = 0 \quad \forall (\boldsymbol{\tau}, \xi) \in \mathbb{K}_h \Big\}\,.$$

The projector Ξ and its discrete counterpart Ξ_h (introduced in [13]) are the key tools in the convergence analysis that we will undertake in the following section. They are characterized by the following properties.

Lemma 4.1. *There exist a linear operators* $\Xi : \mathbb{X} \to \mathbb{X}^{sym}$ *and* $\Xi_h : \mathbb{X} \to \mathbb{X}_h^{sym}$ *such that*

$$\|\Xi(\boldsymbol{\tau}, q)\| + \|\Xi_h(\boldsymbol{\tau}, q)\| \leq C \|(\boldsymbol{\tau}, q)\| \qquad \forall\, (\boldsymbol{\tau}, q) \in \mathbb{X},$$

with $C > 0$*, independent of* λ *and* h*. Moreover,* $\widetilde{\Xi} := \Xi|_{\mathbb{X}^{sym}}$ *is the* $(\cdot,\cdot)_{\mathcal{C}}$*-orthogonal projection of* $\mathbb{X}^{sym}$ *onto* $\mathbb{K}^{\perp}$*.*

Proof. See [13, Section 5]. □

Lemma 4.2. *Assume that* $(\boldsymbol{\tau}, q) \in \mathbb{K}^{\perp}$ *with* $\boldsymbol{\tau} \in [\mathrm{H}^s(\Omega_{\mathrm{S}})]^{d\times d}$ *for some* $s > 0$*, and let* $(\boldsymbol{v}, \boldsymbol{s}) := \mathbf{D}(\boldsymbol{\tau}, q)$ *and* $\boldsymbol{\psi} := \boldsymbol{v}|_{\Sigma}$*. Then, there exists a constant* $C > 0$*, independent of* h *and* λ*, such that*

$$\begin{aligned}\|(\boldsymbol{\tau}, q) - \Xi_h(\boldsymbol{\tau}, q)\| \leq C \Big\{ & \|\boldsymbol{\tau} - \boldsymbol{\Pi}_h\boldsymbol{\tau}\|_{\mathrm{H}(\mathbf{div},\Omega_{\mathrm{S}})} + \|q\boldsymbol{n} - \boldsymbol{\pi}_h(q\boldsymbol{n})\|_{0,\Sigma} + \|\boldsymbol{s} - \boldsymbol{R}_h\boldsymbol{s}\|_{0,\Omega_{\mathrm{S}}} \\ + \quad & \|\boldsymbol{v} - U_h\boldsymbol{v}\|_{0,\Omega_{\mathrm{S}}} + \|\boldsymbol{\psi} - \boldsymbol{\pi}_h\boldsymbol{\psi}\|_{0,\Sigma} + |q - \Pi_h q|_{1,\Omega_{\mathrm{F}}} \Big\}.\end{aligned}$$

Proof. See [13, Lemma 5.8]. □

5 Time-space discretization

5.1 The fully discrete scheme

Given $L \in \mathbb{N}$, we consider a uniform partition of the time interval $[0, T]$ with step size $\Delta t := T/L$. Then, for any continuous function $\phi : [0, T] \to \mathbb{R}$ and for each $k \in \{0, 1, \ldots, L\}$ we denote $\phi^k := \phi(t_k)$, where $t_k := k\,\Delta t$. In addition, we adopt the same notation for vector/tensor valued functions and introduce the notations

$$t_{k+\frac{1}{2}} := \frac{t_{k+1} + t_k}{2}, \quad \phi^{k+\frac{1}{2}} := \frac{\phi^{k+1} + \phi^k}{2}, \quad \phi^{k-\frac{1}{2}} := \frac{\phi^k + \phi^{k-1}}{2},$$

and the discrete time derivatives

$$\partial_t\phi^k := \frac{\phi^{k+1} - \phi^k}{\Delta t} \quad \text{and} \quad \bar{\partial}_t\phi^k := \frac{\phi^k - \phi^{k-1}}{\Delta t},$$

from which we notice that

$$\partial_t\bar{\partial}_t\phi^k = \frac{\bar{\partial}_t\phi^{k+1} - \bar{\partial}_t\phi^k}{\Delta t} = \frac{\partial_t\phi^k - \partial_t\phi^{k-1}}{\Delta t} = \frac{\phi^{k+1} - 2\phi^k + \phi^{k-1}}{\Delta t^2}.$$

The Newmark trapezoidal rule applied to the Galerkin space-semidiscretization introduced in [13] for problem (3.7) reads as follows: For $k = 1, \ldots, L-1$, find $(\boldsymbol{\sigma}_h^{k+1}, p_h^{k+1}) \in \mathbb{X}_h^{\mathrm{sym}}$ such that

$$\begin{aligned}\Big(\partial_t\bar{\partial}_t(\boldsymbol{\sigma}_h^k, p_h^k), (\boldsymbol{\tau}, q)\Big)_{\mathcal{C}} + A\Big(\big(\frac{\boldsymbol{\sigma}_h^{k+\frac{1}{2}} + \boldsymbol{\sigma}_h^{k-\frac{1}{2}}}{2}, \frac{p_h^{k+\frac{1}{2}} + p_h^{k-\frac{1}{2}}}{2}\big), (\boldsymbol{\tau}, q)\Big) \\ = -\rho_{\mathrm{S}}^{-1}\big(\boldsymbol{f}(t_k), \mathbf{div}\,\boldsymbol{\tau}\big)_{0,\Omega_{\mathrm{S}}} \qquad \forall\, (\boldsymbol{\tau}, q) \in \mathbb{X}_h^{\mathrm{sym}}.\end{aligned} \tag{5.1}$$

Moreover, for the sake of simplicity, we assume that the scheme (5.1) is started up with

$$(\boldsymbol{\sigma}_h^0, p_h^0) := \Xi_h(\boldsymbol{\sigma}_0, p_0) \quad \text{and} \quad (\boldsymbol{\sigma}_h^1, p_h^1) := \Xi_h(\boldsymbol{\sigma}(t_1), p(t_1)). \tag{5.2}$$

We insist here upon the fact that it is necessary to introduce a Lagrange multiplier in order to relax the weak symmetry constraint defining $\boldsymbol{\mathcal{W}}_h^{\rm sym}$. This permits one to deal with the well-known BDM-finite element basis functions of the space $\boldsymbol{\mathcal{W}}_h$ in order to obtain the linear systems of equations arising from (5.1) at each iteration step.

Now, recalling that $(\boldsymbol{\sigma}, p)$ stands for the solution of (3.7), we introduce the discrete errors

$$\boldsymbol{e}_{\sigma,h}^k := \boldsymbol{\sigma}_h^*(t_k) - \boldsymbol{\sigma}_h^k \in \boldsymbol{\mathcal{W}}_h^{\rm sym}, \quad \text{and} \qquad e_{p,h}^k := p_h^*(t_k) - p_h^k \in V_h,$$

where, as in [13], we define $(\boldsymbol{\sigma}_h^*, p_h^*) := \Xi_h(\boldsymbol{\sigma}, p)$, and observe that $(\boldsymbol{e}_{\sigma,h}^k, e_{p,h}^k) \in \mathbb{X}_h^{\rm sym}$. Then, thanks to (5.2), we have $\boldsymbol{e}_{\sigma,h}^0 = \boldsymbol{e}_{\sigma,h}^1 = \mathbf{0}$ and $e_{p,h}^0 = e_{p,h}^1 = 0$. In turn, the starting point of our convergence analysis is the following error equation

$$\begin{aligned} &\Big(\partial_t \bar{\partial}_t (\boldsymbol{e}_{\sigma,h}^k, e_{p,h}^k), (\boldsymbol{\tau}, q)\Big)_{\mathcal{C}} \,+\, A\Big((\frac{\boldsymbol{e}_{\sigma,h}^{k+\frac{1}{2}} + \boldsymbol{e}_{\sigma,h}^{k-\frac{1}{2}}}{2}, \frac{e_{p,h}^{k+\frac{1}{2}} + e_{p,h}^{k-\frac{1}{2}}}{2}), (\boldsymbol{\tau}, q)\Big) \\ = \quad &\Big((\boldsymbol{\chi}_{1,\sigma}^k, \boldsymbol{\chi}_{1,p}^k), (\boldsymbol{\tau}, q)\Big)_{\mathcal{C}} \,+\, A\Big((\boldsymbol{\chi}_{2,\sigma}^k, \boldsymbol{\chi}_{2,p}^k), (\boldsymbol{\tau}, q)\Big) \qquad \forall\, (\boldsymbol{\tau}, q) \in \mathbb{X}_h^{\rm sym}\,, \end{aligned} \tag{5.3}$$

where the consistency terms are, for $\xi \in \{\boldsymbol{\sigma}, p\}$,

$$\boldsymbol{\chi}_{1,\xi}^k := \partial_t \bar{\partial}_t \xi_h^*(t_k) \,-\, \ddot{\xi}(t_k) \quad \text{and} \quad \boldsymbol{\chi}_{2,\xi}^k := \frac{\xi_h^*(t_{k+1}) + 2\xi_h^*(t_k) + \xi_h^*(t_{k-1})}{4} - \xi(t_k)\,.$$

By definition of $(\boldsymbol{\sigma}_h^*, p_h^*)$, we have that

$$(\mathbf{div}(\boldsymbol{\sigma}_h^*(t_k) - \boldsymbol{\sigma}(t_k)), \mathbf{div}\,\boldsymbol{\tau})_{0,\Omega_{\rm S}} \,=\, 0 \qquad \forall\, (\boldsymbol{\tau}, q) \in \mathbb{X}_h\,,$$

and

$$(\nabla(p_h^*(t_k) - p(t_k)), \nabla q)_{0,\Omega_{\rm F}} \,=\, (\nabla(p(t_k) - \Pi_h p(t_k)), \nabla q)_{0,\Omega_{\rm F}} \,=\, 0 \qquad \forall\, (\boldsymbol{\tau}, q) \in \mathbb{X}_h\,.$$

Hence, we can substitute in the right hand side of (5.3) the functions $\boldsymbol{\chi}_{2,\sigma}^k$ and $\boldsymbol{\chi}_{2,p}^k$ by

$$\bar{\boldsymbol{\chi}}_{2,\xi}^k := \boldsymbol{\chi}_{2,\xi}^k \,-\, (\xi_h^*(t_k) - \xi(t_k)) \,=\, \frac{\xi_h^*(t_{k+1}) - 2\xi_h^*(t_k) + \xi_h^*(t_{k-1})}{4} \quad \forall\, \xi \in \{\boldsymbol{\sigma}, p\}$$

without altering the error equation.

5.2 Convergence analysis

We begin by establishing the following stability result.

Lemma 5.1. *There exists a constant $C > 0$, independent of λ, h and Δt, such that for each $n \in \mathbb{N}$ there holds*

$$\begin{aligned} &\max_n \big\|(\partial_t \boldsymbol{e}_{\sigma,h}^n, \partial_t e_{p,h}^n)\big\|_{0,\mathcal{C}} \,+\, \max_n \Big\|\mathbf{div}\,\boldsymbol{e}_{\sigma,h}^{n+\frac{1}{2}}\Big\|_{0,\Omega_{\rm S}} \,+\, \max_n \Big\|\nabla e_{p,h}^{n+\frac{1}{2}}\Big\|_{0,\Omega_{\rm F}} \\ &\quad \le\, C\Big\{ \max_n \big\|(\boldsymbol{\chi}_{1,\sigma}^n, \boldsymbol{\chi}_{1,p}^n)\big\|_{0,\mathcal{C}} \,+\, \max_n \big\|\mathbf{div}\,\partial_t \bar{\boldsymbol{\chi}}_{2,\sigma}^n\big\|_{0,\Omega_{\rm S}} \,+\, \max_n \big\|\nabla \partial_t \bar{\boldsymbol{\chi}}_{2,p}^n\big\|_{0,\Omega_{\rm F}} \\ &\quad + \max_n \big\|\mathbf{div}\,\bar{\boldsymbol{\chi}}_{2,\sigma}^n\big\|_{0,\Omega_{\rm S}} \,+\, \max_n \big\|\nabla \bar{\boldsymbol{\chi}}_{2,p}^n\big\|_{0,\Omega_{\rm F}} \Big\}\,. \end{aligned} \tag{5.4}$$

Proof. Taking $(\boldsymbol{\tau}, q) = (\dfrac{\boldsymbol{e}_{\sigma,h}^{k+1} - \boldsymbol{e}_{\sigma,h}^{k-1}}{2\Delta t}, \dfrac{e_{p,h}^{k+1} - e_{p,h}^{k-1}}{2\Delta t})$ in (5.3) and using

$$\frac{\boldsymbol{e}_{\sigma,h}^{k+1} - \boldsymbol{e}_{\sigma,h}^{k-1}}{2\Delta t} \;=\; \frac{\boldsymbol{e}_{\sigma,h}^{k+\frac{1}{2}} - \boldsymbol{e}_{\sigma,h}^{k-\frac{1}{2}}}{\Delta t} \;=\; \frac{\partial_t \boldsymbol{e}_{\sigma,h}^{k} + \partial_t \boldsymbol{e}_{\sigma,h}^{k-1}}{2}\,,$$

and the similar identity for $\dfrac{e_{p,h}^{k+1} - e_{p,h}^{k-1}}{2\Delta t}$, we find that

$$\begin{aligned}
&\frac{1}{2\Delta t}\Big((\partial_t \boldsymbol{e}_{\sigma,h}^{k} - \partial_t \boldsymbol{e}_{\sigma,h}^{k-1}, \partial_t e_{p,h}^{k} - \partial_t e_{p,h}^{k-1}), (\partial_t \boldsymbol{e}_{\sigma,h}^{k} + \partial_t \boldsymbol{e}_{\sigma,h}^{k-1}, \partial_t e_{p,h}^{k} + \partial_t e_{p,h}^{k-1})\Big)_{0,\mathcal{C}} \\
+\;& \frac{1}{2\Delta t} A\Big((\boldsymbol{e}_{\sigma,h}^{k+\frac{1}{2}} + \boldsymbol{e}_{\sigma,h}^{k-\frac{1}{2}}, e_{p,h}^{k+\frac{1}{2}} + e_{p,h}^{k-\frac{1}{2}}), (\boldsymbol{e}_{\sigma,h}^{k+\frac{1}{2}} - \boldsymbol{e}_{\sigma,h}^{k-\frac{1}{2}}, e_{p,h}^{k+\frac{1}{2}} - e_{p,h}^{k-\frac{1}{2}})\Big) \\
=\;& \Big((\boldsymbol{\chi}_{1,\sigma}^{k}, \boldsymbol{\chi}_{1,p}^{k}), (\frac{\partial_t \boldsymbol{e}_{\sigma,h}^{k} + \partial_t \boldsymbol{e}_{\sigma,h}^{k-1}}{2}, \frac{\partial_t e_{p,h}^{k} + \partial_t e_{p,h}^{k-1}}{2})\Big)_{0,\mathcal{C}} \\
+\;& A\Big((\bar{\boldsymbol{\chi}}_{2,\sigma}^{k}, \bar{\boldsymbol{\chi}}_{2,p}^{k}), (\frac{\boldsymbol{e}_{\sigma,h}^{k+\frac{1}{2}} - \boldsymbol{e}_{\sigma,h}^{k-\frac{1}{2}}}{\Delta t}, \frac{e_{p,h}^{k+\frac{1}{2}} - e_{p,h}^{k-\frac{1}{2}}}{\Delta t})\Big)\,,
\end{aligned}$$

which can also be written as

$$\begin{aligned}
&\frac{1}{2\Delta t}\Big(\Big\|(\partial_t \boldsymbol{e}_{\sigma,h}^{k}, \partial_t e_{p,h}^{k})\Big\|_{0,\mathcal{C}}^2 \;-\; \Big\|(\partial_t \boldsymbol{e}_{\sigma,h}^{k-1}, \partial_t e_{p,h}^{k-1})\Big\|_{0,\mathcal{C}}^2 \Big) \\
+\;& \frac{1}{2\Delta t}\Big\{ A\Big((\boldsymbol{e}_{\sigma,h}^{k+\frac{1}{2}}, e_{p,h}^{k+\frac{1}{2}}), (\boldsymbol{e}_{\sigma,h}^{k+\frac{1}{2}}, e_{p,h}^{k+\frac{1}{2}})\Big) \;-\; A\Big((\boldsymbol{e}_{\sigma,h}^{k-\frac{1}{2}}, e_{p,h}^{k-\frac{1}{2}}), (\boldsymbol{e}_{\sigma,h}^{k-\frac{1}{2}}, e_{p,h}^{k-\frac{1}{2}})\Big)\Big\} \\
=\;& \Big((\boldsymbol{\chi}_{1,\sigma}^{k}, \boldsymbol{\chi}_{1,p}^{k}), (\frac{\partial_t \boldsymbol{e}_{\sigma,h}^{k} + \partial_t \boldsymbol{e}_{\sigma,h}^{k-1}}{2}, \frac{\partial_t e_{p,h}^{k} + \partial_t e_{p,h}^{k-1}}{2})\Big)_{0,\mathcal{C}} \\
+\;& A\Big((\bar{\boldsymbol{\chi}}_{2,\sigma}^{k}, \bar{\boldsymbol{\chi}}_{2,p}^{k}), (\frac{\boldsymbol{e}_{\sigma,h}^{k+\frac{1}{2}} - \boldsymbol{e}_{\sigma,h}^{k-\frac{1}{2}}}{\Delta t}, \frac{e_{p,h}^{k+\frac{1}{2}} - e_{p,h}^{k-\frac{1}{2}}}{\Delta t})\Big)\,.
\end{aligned}$$

In this way, multiplying by $2\Delta t$ and summing up the foregoing identity over $k = 1, \ldots, n$, gives

$$\begin{aligned}
&\big\|(\partial_t \boldsymbol{e}_{\sigma,h}^{n}, \partial_t e_{p,h}^{n})\big\|_{0,\mathcal{C}}^2 \;+\; A\Big((\boldsymbol{e}_{\sigma,h}^{n+\frac{1}{2}}, e_{p,h}^{n+\frac{1}{2}}), (\boldsymbol{e}_{\sigma,h}^{n+\frac{1}{2}}, e_{p,h}^{n+\frac{1}{2}})\Big) \\
=\;& 2\Delta t \sum_{k=1}^{n} \Big((\boldsymbol{\chi}_{1,\sigma}^{k}, \boldsymbol{\chi}_{1,p}^{k}), (\frac{\partial_t \boldsymbol{e}_{\sigma,h}^{k} + \partial_t \boldsymbol{e}_{\sigma,h}^{k-1}}{2}, \frac{\partial_t e_{p,h}^{k} + \partial_t e_{p,h}^{k-1}}{2})\Big)_{0,\mathcal{C}} \\
+\;& 2\Delta t \sum_{k=1}^{n} A\Big((\bar{\boldsymbol{\chi}}_{2,\sigma}^{k}, \bar{\boldsymbol{\chi}}_{2,p}^{k}), (\frac{\boldsymbol{e}_{\sigma,h}^{k+\frac{1}{2}} - \boldsymbol{e}_{\sigma,h}^{k-\frac{1}{2}}}{\Delta t}, \frac{e_{p,h}^{k+\frac{1}{2}} - e_{p,h}^{k-\frac{1}{2}}}{\Delta t})\Big) \\
=\;& 2\Delta t \sum_{k=1}^{n} \Big((\boldsymbol{\chi}_{1,\sigma}^{k}, \boldsymbol{\chi}_{1,p}^{k}), (\frac{\partial_t \boldsymbol{e}_{\sigma,h}^{k} + \partial_t \boldsymbol{e}_{\sigma,h}^{k-1}}{2}, \frac{\partial_t e_{p,h}^{k} + \partial_t e_{p,h}^{k-1}}{2})\Big)_{0,\mathcal{C}} \\
-\;& 2\,\Delta t \sum_{k=1}^{n-1} A\Big((\partial_t \bar{\boldsymbol{\chi}}_{2,\sigma}^{k}, \partial_t \bar{\boldsymbol{\chi}}_{2,p}^{k}), (\boldsymbol{e}_{\sigma,h}^{k+\frac{1}{2}}, e_{p,h}^{k+\frac{1}{2}})\Big) \;+\; 2A\Big((\bar{\boldsymbol{\chi}}_{2,\sigma}^{n}, \bar{\boldsymbol{\chi}}_{2,p}^{n}), (\boldsymbol{e}_{\sigma,h}^{n+\frac{1}{2}}, e_{p,h}^{n+\frac{1}{2}})\Big)\,.
\end{aligned}$$

It is now straightforward to deduce from the last identity and the Cauchy-Schwarz inequality, that there exists a constant $C_0 > 0$, independent of λ, h and Δt, such that

$$\max_n \big\|(\partial_t \boldsymbol{e}^n_{\sigma,h}, \partial_t e^n_{p,h}\big\|_{0,\mathcal{C}} + \max_n \sqrt{A\Big((\boldsymbol{e}^{n+\frac12}_{\sigma,h}, e^{n+\frac12}_{p,h}), (\boldsymbol{e}^{n+\frac12}_{\sigma,h}, e^{n+\frac12}_{p,h})\Big)}$$
$$\leq \quad C_0 \Big\{ \Delta t \sum_{k=1}^{L} \Big\|(\boldsymbol{\chi}^k_{1,\sigma}, \boldsymbol{\chi}^k_{1,p})\Big\|_{0,\mathcal{C}} + \Delta t \sum_{k=1}^{L-1} \sqrt{A\Big((\partial_t \bar{\boldsymbol{\chi}}^k_{2,\sigma}, \partial_t \bar{\boldsymbol{\chi}}^k_{2,p}), (\partial_t \bar{\boldsymbol{\chi}}^k_{2,\sigma}, \partial_t \bar{\boldsymbol{\chi}}^k_{2,p})\Big)}$$
$$+ \quad \max_n \sqrt{A\Big((\bar{\boldsymbol{\chi}}^n_{2,\sigma}, \bar{\boldsymbol{\chi}}^n_{2,p}), (\bar{\boldsymbol{\chi}}^n_{2,\sigma}, \bar{\boldsymbol{\chi}}^n_{2,p})\Big)} \Big\},$$

and the result follows from the lower bound of (3.8). □

We now aim to bound the expression

$$\mathcal{M}_h(\boldsymbol{\sigma}, p) := \max_n \Big\|(\dot{\boldsymbol{\sigma}}, \dot{p})(t_{n+\frac12}) - (\partial_t \boldsymbol{\sigma}^n_h, \partial_t p^n_h)\Big\|_{0,\mathcal{C}} + \max_n \Big\|\mathbf{div}\, \boldsymbol{\sigma}(t_{n+\frac12}) - \mathbf{div}\, \boldsymbol{\sigma}_h^{n+\frac12}\Big\|_{0,\Omega_{\mathrm{S}}}$$
$$+ \quad \max_n \Big\|\nabla\big(p(t_{n+\frac12}) - p_h^{n+\frac12}\big)\Big\|_{0,\Omega_{\mathrm{F}}}$$

To this end, we first observe thanks to the triangle inequality and the stability estimate (5.4) that

$$\mathcal{M}_h(\boldsymbol{\sigma}, p) \leq \widetilde{\mathcal{M}}_h(\boldsymbol{\sigma}, p) + C\, \widehat{\mathcal{M}}_h(\boldsymbol{\sigma}, p)\,, \tag{5.5}$$

where

$$\widetilde{\mathcal{M}}_h(\boldsymbol{\sigma}, p) := \max_n \Big\|(\dot{\boldsymbol{\sigma}}, \dot{p})(t_{n+\frac12}) - (\partial_t \boldsymbol{\sigma}^*_h(t_n), \partial_t p^*_h(t_n))\Big\|_{0,\mathcal{C}}$$
$$+ \quad \max_n \big\| \mathbf{div}\, \big(\boldsymbol{\sigma}(t_{n+\frac12}) - (\boldsymbol{\sigma}^*_h)^{n+\frac12}\big)\big\|_{0,\Omega_{\mathrm{S}}} + \max_n \Big\|\nabla\big(p(t_{n+\frac12}) - (p^*_h)^{n+\frac12}\big)\Big\|_{0,\Omega_{\mathrm{F}}}$$

and

$$\widehat{\mathcal{M}}_h(\boldsymbol{\sigma}, p) := \max_n \big\|(\boldsymbol{\chi}^n_{1,\sigma}, \boldsymbol{\chi}^n_{1,p})\big\|_{0,\mathcal{C}} + \max_n \big\|\mathbf{div}\, \partial_t \bar{\boldsymbol{\chi}}^n_{2,\sigma}\big\|_{0,\Omega_{\mathrm{S}}}$$
$$+ \quad \max_n \big\|\nabla \partial_t \bar{\boldsymbol{\chi}}^n_{2,p}\big\|_{0,\Omega_{\mathrm{F}}} + \max_n \big\|\mathbf{div}\, \bar{\boldsymbol{\chi}}^n_{2,\sigma}\big\|_{0,\Omega_{\mathrm{S}}} + \max_n \big\|\nabla \bar{\boldsymbol{\chi}}^n_{2,p}\big\|_{0,\Omega_{\mathrm{F}}}\,.$$

The following two lemmas apply Taylor expansions with integral remainder to derive upper bounds for the terms on the right hand side of (5.5).

Lemma 5.2. *Assume that the solution* $(\boldsymbol{\sigma}, p) \in \mathcal{C}^0((0,T); \mathbb{X}^{sym}) \cap \mathcal{C}^1((0,T); \mathbb{H}^{sym})$ *to problem* (3.7) *satisfies* $\boldsymbol{\sigma} \in \mathcal{C}^2((0,T); \mathbb{H}(\mathbf{div}, \Omega_{\mathrm{S}}) \cap H^s(\Omega_{\mathrm{S}})^{n\times n}) \cap \mathcal{C}^3((0,T); \mathbb{H}(\mathbf{div}, \Omega_{\mathrm{S}}))$ *for some* $s > 0$ *and* $p \in \mathcal{C}^3(H^1(\Omega_{\mathrm{F}}))$. *Then, there exists a constant* $C > 0$, *independent of* λ, h *and* Δt, *such that*

$$\widetilde{\mathcal{M}}_h(\boldsymbol{\sigma}, p) \leq C \Big\{ \|\boldsymbol{\sigma} - \boldsymbol{\sigma}^*_h\|_{\mathrm{W}^{1,\infty}((0,T);\mathrm{H}(\mathbf{div},\Omega_{\mathrm{S}}))} + \|p - p^*_h\|_{\mathrm{W}^{1,\infty}((0,T);H^1(\Omega_{\mathrm{F}}))}$$
$$+ \quad (\Delta t)^2 \Big(\|\boldsymbol{\sigma}^*_h\|_{\mathrm{W}^{3,\infty}((0,T);\mathrm{H}(\mathbf{div},\Omega_{\mathrm{S}}))} + \|p^*_h\|_{\mathrm{W}^{3,\infty}((0,T);H^1(\Omega_{\mathrm{F}}))} \Big) \Big\}. \tag{5.6}$$

Proof. Using Taylor expansions centered at $t = t_{n+\frac12}$ gives for each $\xi \in \{\boldsymbol{\sigma}, p\}$,

$$\xi(t_{n+\frac12}) - (\xi^*_h)^{n+\frac12} = \xi(t_{n+\frac12}) - \xi^*_h(t_{n+\frac12}) - \frac12 \int_{t_n}^{t_{n+1}} \ddot{\xi}^*_h(t) \Big(\frac{\Delta t}{2} - |t - t_{n+\frac12}|\Big)\, \mathrm{d}t \tag{5.7}$$

and

$$\dot{\xi}(t_{n+\frac{1}{2}}) - \partial_t \xi_h^*(t_n) \,=\, \dot{\xi}(t_{n+\frac{1}{2}}) - \dot{\xi}_h^*(t_{n+\frac{1}{2}}) \,-\, \frac{1}{2\Delta t}\int_{t_{n+\frac{1}{2}}}^{t_{n+1}} \frac{\mathrm{d}^3\xi_h^*(t)}{\mathrm{d}t^3}(t_{n+1}-t)^2\,\mathrm{d}t \\ -\,\frac{1}{2\Delta t}\int_{t_n}^{t_{n+\frac{1}{2}}} \frac{\mathrm{d}^3\xi_h^*(t)}{\mathrm{d}t^3}(t_n-t)^2\,\mathrm{d}t\,. \tag{5.8}$$

Then, it is not difficult to see that using (5.8) with $\xi = \boldsymbol{\sigma}$ and $\xi = p$, and then applying the space differential operators **div** and ∇ to $\xi = \boldsymbol{\sigma}$ and $\xi = p$, respectively, in (5.7), we arrive at (5.6). □

Lemma 5.3. *Assume that the solution* $(\boldsymbol{\sigma}, p) \in \mathcal{C}^0((0,T);\mathbb{X}^{sym}) \cap \mathcal{C}^1((0,T);\mathbb{H}^{sym})$ *to problem* (3.7) *satisfies* $\boldsymbol{\sigma} \in \mathcal{C}^2((0,T);\mathbb{H}(\mathbf{div},\Omega_{\mathrm{S}}) \cap H^s(\Omega_{\mathrm{S}})^{n\times n}) \cap \mathcal{C}^4((0,T);\mathbb{H}(\mathbf{div},\Omega_{\mathrm{S}}))$ *for some* $s > 0$ *and* $p \in \mathcal{C}^4(H^1(\Omega_{\mathrm{F}}))$. *Then, there exists a constant* $C > 0$, *independent of* λ, h *and* Δt, *such that*

$$\widehat{\mathcal{M}}_h(\boldsymbol{\sigma}, p) \,\leq\, C\Big\{\, \|\boldsymbol{\sigma} - \boldsymbol{\sigma}_h^*\|_{\mathrm{W}^{2,\infty}((0,T);\mathrm{H}(\mathbf{div},\Omega_{\mathrm{S}}))} \,+\, \|p - p_h^*\|_{\mathrm{W}^{2,\infty}((0,T);H^1(\Omega_{\mathrm{F}}))} \\ +\quad (\Delta t)^2\Big(\, \|\boldsymbol{\sigma}_h^*\|_{\mathrm{W}^{4,\infty}((0,T);\mathrm{H}(\mathbf{div},\Omega_{\mathrm{S}}))} \,+\, \|p_h^*\|_{\mathrm{W}^{4,\infty}((0,T);H^1(\Omega_{\mathrm{F}}))}\,\Big)\Big\}\,. \tag{5.9}$$

Proof. Using now Taylor expansions centered at $t = t_n$ we have for each $\xi \in \{\boldsymbol{\sigma}, p\}$,

$$\boldsymbol{\chi}_{1,\xi}^n = \ddot{\xi}_h^*(t_n) - \ddot{\xi}(t_n) + \,\frac{1}{6(\Delta t)^2}\int_{t_{n-1}}^{t_{n+1}} \frac{\mathrm{d}^4\xi_h^*(t)}{\mathrm{d}t^4}\,(\Delta t - |t-t_n|)^3\,\mathrm{d}t\,, \tag{5.10}$$

$$\bar{\boldsymbol{\chi}}_{2,\xi}^n = \frac{1}{4}\int_{t_{n-1}}^{t_{n+1}} \ddot{\xi}_h^*(t)(\Delta t - |t-t_n|)\,\mathrm{d}t\,, \tag{5.11}$$

and

$$\partial_t \bar{\boldsymbol{\chi}}_{2,\xi}^n = \frac{\xi_h^*(t_{n+2}) - 3\xi_h^*(t_{n+1}) + 3\xi_h^*(t_n) - \xi_h^*(t_{n-1})}{4\Delta t} \,=\, \frac{1}{8\Delta t}\left\{\int_{t_n}^{t_{n+2}} \frac{\mathrm{d}^3\xi_h^*(t)}{\mathrm{d}t^3}(t_{n+2}-t)^2\,\mathrm{d}t \right. \\ \left. -\,3\int_{t_n}^{t_{n+1}} \frac{\mathrm{d}^3\xi_h^*(t)}{\mathrm{d}t^3}(t_{n+1}-t)^2\,\mathrm{d}t \,+\, \int_{t_{n-1}}^{t_n} \frac{\mathrm{d}^3\xi_h^*(t)}{\mathrm{d}t^3}(t_{n-1}-t)^2\,\mathrm{d}t\right\}. \tag{5.12}$$

In this way, proceeding similarly as for the previous lemma, that is by applying now (5.10), (5.11) and (5.12), we obtain (5.9) Further details are omitted. □

As a consequence of Lemmas 5.2, 5.3, and 4.2, we are able to establish next the required bound for $\mathcal{M}_h(\boldsymbol{\sigma}, p)$.

Lemma 5.4. *Assume that the solution* $(\boldsymbol{\sigma}, p) \in \mathcal{C}^0((0,T);\mathbb{X}^{sym}) \cap \mathcal{C}^1((0,T);\mathbb{H}^{sym})$ *to problem* (3.7) *satisfies* $\boldsymbol{\sigma} \in \mathcal{C}^2((0,T);\mathbb{H}(\mathbf{div},\Omega_{\mathrm{S}}) \cap H^s(\Omega_{\mathrm{S}})^{n\times n}) \cap \mathcal{C}^4((0,T);\mathbb{H}(\mathbf{div},\Omega_{\mathrm{S}}))$ *for some* $s > 0$ *and* $p \in \mathcal{C}^4(H^1(\Omega_{\mathrm{F}}))$. *Then, there exists a constant* $C > 0$, *independent of* λ, h *and* Δt, *such that*

$$\mathcal{M}_h(\boldsymbol{\sigma}, p) \,\leq\, C\Big\{\, \|\boldsymbol{\sigma} - \boldsymbol{\Pi}_h\boldsymbol{\sigma}\|_{\mathrm{W}^{2,\infty}((0,T);\mathbf{H}(\mathbf{div},\Omega_{\mathrm{S}}))} \,+\, \|p\boldsymbol{n} - \boldsymbol{\pi}_h(p\boldsymbol{n})\|_{\mathrm{W}^{2,\infty}((0,T);\mathrm{L}^2(\Sigma)^d)} \\ +\quad \|\boldsymbol{r} - \boldsymbol{R}_h\boldsymbol{r}\|_{\mathrm{W}^{2,\infty}((0,T);[\mathrm{L}^2(\Omega_{\mathrm{S}})]^{d\times d}_{skew})} \,+\, \|\boldsymbol{u} - U_h\boldsymbol{u}\|_{\mathrm{W}^{2,\infty}((0,T);\mathrm{L}^2(\Omega_{\mathrm{S}})^d)} \\ +\quad \|\boldsymbol{\psi} - \boldsymbol{\pi}_h\boldsymbol{\psi}\|_{\mathrm{W}^{2,\infty}((0,T);\mathrm{L}^2(\Sigma)^d)} \,+\, \|\nabla(p - \Pi_h p)\|_{\mathrm{W}^{2,\infty}((0,T);\mathrm{L}^2(\Omega_{\mathrm{F}})^d)} \\ +\quad (\Delta t)^2\Big(\, \|\boldsymbol{\sigma}\|_{\mathrm{W}^{4,\infty}((0,T);\mathrm{H}(\mathbf{div},\Omega_{\mathrm{S}}))} \,+\, \|p\|_{\mathrm{W}^{4,\infty}((0,T);H^1(\Omega_{\mathrm{F}}))}\,\Big)\Big\}\,, \tag{5.13}$$

where $(\boldsymbol{u}, \boldsymbol{r}) = \mathbf{D}(\boldsymbol{\sigma}, p)$ *and* $\boldsymbol{\psi} = \boldsymbol{u}|_\Sigma$.

Proof. It follows straightforwardly from the initial estimate (5.5) and Lemmas 5.2 and 5.3 that

$$\begin{aligned}\mathcal{M}_h(\boldsymbol{\sigma},p) \leq C\Big\{ &\|\boldsymbol{\sigma}-\boldsymbol{\sigma}_h^*\|_{\mathrm{W}^{2,\infty}((0,T);\mathrm{H}(\mathbf{div},\Omega_{\mathrm{S}}))} + \|p-p_h^*\|_{\mathrm{W}^{2,\infty}((0,T);H^1(\Omega_{\mathrm{F}}))} \\ &+(\Delta t)^2\Big(\|\boldsymbol{\sigma}_h^*\|_{\mathrm{W}^{4,\infty}((0,T);\mathrm{H}(\mathbf{div},\Omega_{\mathrm{S}}))} + \|p_h^*\|_{\mathrm{W}^{4,\infty}((0,T);H^1(\Omega_{\mathrm{F}}))}\Big)\Big\},\end{aligned} \tag{5.14}$$

On the other hand, the uniform boundedness of $\Xi_h : \mathbb{X} \to \mathbb{X}_h^{\mathrm{sym}}$ with respect to h and λ, and our regularity assumptions, imply that there exists a constant $C>0$, independent of h and λ, such that

$$\begin{aligned}&\|\boldsymbol{\sigma}_h^*\|_{\mathrm{W}^{4,\infty}((0,T);\mathrm{H}(\mathbf{div},\Omega_{\mathrm{S}}))} + \|p_h^*\|_{\mathrm{W}^{4,\infty}((0,T);H^1(\Omega_{\mathrm{F}}))} \\ &\quad\leq C\Big\{\|\boldsymbol{\sigma}\|_{\mathrm{W}^{4,\infty}((0,T);\mathrm{H}(\mathbf{div},\Omega_{\mathrm{S}}))} + \|p\|_{\mathrm{W}^{4,\infty}((0,T);H^1(\Omega_{\mathrm{F}}))}\Big\}.\end{aligned} \tag{5.15}$$

Finally, combining (5.14) and (5.15) we conclude that

$$\begin{aligned}\mathcal{M}_h(\boldsymbol{\sigma},p) \leq\ C\Big\{ &\|\boldsymbol{\sigma}-\boldsymbol{\sigma}_h^*\|_{\mathrm{W}^{2,\infty}((0,T);\mathrm{H}(\mathbf{div},\Omega_{\mathrm{S}}))} + \|p-p_h^*\|_{\mathrm{W}^{2,\infty}((0,T);H^1(\Omega_{\mathrm{F}}))} \\ &+\ (\Delta t)^2\Big(\|\boldsymbol{\sigma}\|_{\mathrm{W}^{4,\infty}((0,T);\mathrm{H}(\mathbf{div},\Omega_{\mathrm{S}}))} + \|p\|_{\mathrm{W}^{4,\infty}((0,T);H^1(\Omega_{\mathrm{F}}))}\Big)\Big\},\end{aligned}$$

and the result follows by applying Lemma 4.2 to $(\boldsymbol{\sigma},p)\in\mathbb{K}^{\perp}$. ∎

We notice here that while the constant $C>0$ appearing in (5.13) is independent of λ, the first error term on the left-hand side, namely $(\dot{\boldsymbol{\sigma}},\dot{p})(t_{n+\frac{1}{2}}) - (\partial_t\boldsymbol{\sigma}_h^n, \partial_t p_h^n)$, is estimated in the λ-dependent norm $\|\cdot\|_{\mathcal{C}}$. Hence, Lemma 5.4 ensures that only the convergence of the semi-norms

$$\max_n \left\|\mathbf{div}\left(\boldsymbol{\sigma}(t_{n+\frac{1}{2}}) - \boldsymbol{\sigma}_h^{n+\frac{1}{2}}\right)\right\|_{0,\Omega_{\mathrm{S}}} \quad\text{and}\quad \max_n \left\|\nabla\big(p(t_{n+\frac{1}{2}}) - p_h^{n+\frac{1}{2}}\big)\right\|_{0,\Omega_{\mathrm{F}}}$$

remain unaltered when λ goes to infinity. We aim now to apply Lemma 3.1 to deduce the same stability behaviour in the full $\mathbb{X}$-norm. To this end, we first need the following intermediate result.

Lemma 5.5. *Under the hypotheses of Lemma 5.4 there exists a constant $C>0$, independent of λ, h and Δt, such that*

$$\begin{aligned}&\max_n \left\|(\boldsymbol{\sigma},p)(t_{n+\frac{1}{2}}) - (\boldsymbol{\sigma}_h^{n+\frac{1}{2}}, p_h^{n+\frac{1}{2}})\right\|_{0,\mathcal{C}} \\ &\quad\leq\quad C\Big\{\|\boldsymbol{\sigma}-\boldsymbol{\Pi}_h\boldsymbol{\sigma}\|_{\mathrm{W}^{2,\infty}((0,T);\mathbf{H}(\mathbf{div},\Omega_{\mathrm{S}}))} + \|\nabla(p-\Pi_h p)\|_{\mathrm{W}^{2,\infty}((0,T);\mathrm{L}^2(\Omega_{\mathrm{F}})^d)} \\ &\quad+\quad \|p\boldsymbol{n}-\boldsymbol{\pi}_h(p\boldsymbol{n})\|_{\mathrm{W}^{2,\infty}((0,T);\mathrm{L}^2(\Sigma)^d)} + \|\boldsymbol{u}-U_h\boldsymbol{u}\|_{\mathrm{W}^{2,\infty}((0,T);\mathrm{L}^2(\Omega_{\mathrm{S}})^d)} \\ &\quad+\quad \|\boldsymbol{\psi}-\boldsymbol{\pi}_h\boldsymbol{\psi}\|_{\mathrm{W}^{2,\infty}((0,T);\mathrm{L}^2(\Sigma)^d)} + \|\boldsymbol{r}-\boldsymbol{R}_h\boldsymbol{r}\|_{\mathrm{W}^{2,\infty}((0,T);[\mathrm{L}^2(\Omega_{\mathrm{S}})]^{d\times d}_{skew})} \\ &\quad+\quad (\Delta t)^2\Big(\|\boldsymbol{\sigma}\|_{\mathrm{W}^{4,\infty}((0,T);\mathrm{H}(\mathbf{div},\Omega_{\mathrm{S}}))} + \|p\|_{\mathrm{W}^{4,\infty}((0,T);H^1(\Omega_{\mathrm{F}}))}\Big)\Big\}.\end{aligned} \tag{5.16}$$

Proof. We first notice that for each $\xi\in\{\boldsymbol{\sigma},p\}$ there holds

$$\begin{aligned}(\xi(t_{k+\frac{1}{2}}) - \xi_h^{k+\frac{1}{2}}) - (\xi(t_{k-\frac{1}{2}}) - \xi_h^{k-\frac{1}{2}}) &= \xi(t_{k+\frac{1}{2}}) - \xi(t_{k-\frac{1}{2}}) - \frac{\Delta t}{2}(\dot{\xi}(t_{k+\frac{1}{2}}) + \dot{\xi}(t_{k-\frac{1}{2}})) \\ &+\quad \frac{\Delta t}{2}(\dot{\xi}(t_{k+\frac{1}{2}}) - \partial_t\xi_h^k) + \frac{\Delta t}{2}\big(\dot{\xi}(t_{k-\frac{1}{2}}) - \partial_t\xi_h^{k-1}\big).\end{aligned} \tag{5.17}$$

Then, using a Taylor expansion centered at $t = t_k$, we find that

$$\begin{aligned}\xi(t_{k+\frac{1}{2}}) - \xi(t_{k-\frac{1}{2}}) - \frac{\Delta t}{2}(\dot\xi(t_{k+\frac{1}{2}}) + \dot\xi(t_{k-\frac{1}{2}})) = \frac{1}{2}\int_{t_k}^{t_{k+\frac{1}{2}}} \frac{\mathrm{d}^3\xi(t)}{\mathrm{d}t^3}(t_{k+\frac{1}{2}} - t)^2\,\mathrm{d}t \\ + \frac{1}{2}\int_{t_{k-\frac{1}{2}}}^{t_k} \frac{\mathrm{d}^3\xi(t)}{\mathrm{d}t^3}(t_{k-\frac{1}{2}} - t)^2\,\mathrm{d}t \;-\; \frac{\Delta t}{2}\int_{t_{k-\frac{1}{2}}}^{t_{k+\frac{1}{2}}} \frac{\mathrm{d}^3\xi(t)}{\mathrm{d}t^3}(\frac{\Delta t}{2} - |t - t_k|)\,\mathrm{d}t \quad \forall\, \xi \in \{\boldsymbol{\sigma}, p\}\,.\end{aligned} \tag{5.18}$$

Substituting (5.18) in (5.17), and summing the resulting identities over $k = 1, \ldots, n$, we deduce that there exists a constant $C_0 > 0$, independent of λ, h and Δt, such that

$$\begin{aligned}\max_n \left\|(\boldsymbol{\sigma}, p)(t_{n+\frac{1}{2}}) - (\boldsymbol{\sigma}_h^{n+\frac{1}{2}}, p_h^{n+\frac{1}{2}})\right\|_{0,\mathcal{C}} \le\; C_0 \Big\{ (\Delta t)^2 \big(\|\boldsymbol{\sigma}\|_{\mathrm{W}^{3,\infty}((0,T);\mathrm{L}^2(\Omega_\mathrm{S})^{d\times d})} \\ +\; \|p\|_{\mathrm{W}^{3,\infty}((0,T);\mathrm{L}^2(\Omega_\mathrm{F}))}\big) \qquad + \max_n \left\|(\dot{\boldsymbol{\sigma}}, \dot p)(t_{n+\frac{1}{2}}) - (\partial_t\boldsymbol{\sigma}_h^n, \partial_t p_h^n)\right\|_{0,\mathcal{C}} \Big\}\,.\end{aligned}$$

Finally, (5.16) is a direct consequence of the foregoing estimate and Lemma 5.4. □

We are now in a position to establish the following asymptotic error estimate.

Theorem 5.1. *Assume that the solutions $(\boldsymbol{\sigma}, p)$ to problem (3.7) satisfies the regularity assumptions $(\boldsymbol{\sigma}, p) \in \mathcal{C}^4((0,T); \mathbb{X}^{sym})$ and $(\boldsymbol{u}, p) \in \mathcal{C}^2\Big((0,T); \mathrm{H}^{k+1}(\Omega_\mathrm{S})^d \times \mathrm{H}^{k+1}(\Omega_\mathrm{F})\Big)$, for some $k \ge 1$, where $\boldsymbol{u}$ is the displacement associated to $(\boldsymbol{\sigma}, p)$ through operator D. Then, there exists a constant $C > 0$, independent of λ, h and Δt, such that*

$$\max_n \left\|\Big(\boldsymbol{\sigma}(t_{n+\frac{1}{2}}) - \boldsymbol{\sigma}_h^{n+\frac{1}{2}}, p(t_{n+\frac{1}{2}}) - p_h^{n+\frac{1}{2}}\Big)\right\| \le\; C\Big\{ h^k + (\Delta t)^2) \Big\}\,.$$

Proof. We deduce immediately from Lemmas 5.4 and 5.5 that there exists a constant $C_0 > 0$, independent of λ, h and Δt, such that

$$\begin{aligned}&\max_n \left\|(\boldsymbol{\sigma}, p)(t_{n+\frac{1}{2}}) - (\boldsymbol{\sigma}_h^{n+\frac{1}{2}}, p_h^{n+\frac{1}{2}})\right\|_{0,\mathcal{C}} + \max_n \left\|\mathbf{div}\left(\boldsymbol{\sigma}(t_{n+\frac{1}{2}}) - \boldsymbol{\sigma}_h^{n+\frac{1}{2}}\right)\right\|_{0,\Omega_\mathrm{S}} \\ &+ \max_n \left\|\nabla\big(p(t_{n+\frac{1}{2}}) - p_h^{n+\frac{1}{2}}\big)\right\|_{0,\Omega_\mathrm{F}} \le\; C_0 \Big\{ \|\boldsymbol{\sigma} - \boldsymbol{\Pi}_h\boldsymbol{\sigma}\|_{\mathrm{W}^{2,\infty}((0,T);\mathbf{H}(\mathbf{div},\Omega_\mathrm{S}))} \\ &\quad + \quad \|p\boldsymbol{n} - \boldsymbol{\pi}_h(p\boldsymbol{n})\|_{\mathrm{W}^{2,\infty}((0,T);\mathrm{L}^2(\Sigma)^d)} + \|\boldsymbol{r} - \boldsymbol{R}_h\boldsymbol{r}\|_{\mathrm{W}^{2,\infty}((0,T);[\mathrm{L}^2(\Omega_\mathrm{S})]^{d\times d}_{\mathrm{skew}})} \\ &\quad + \quad \|\boldsymbol{u} - U_h\boldsymbol{u}\|_{\mathrm{W}^{2,\infty}((0,T);\mathrm{L}^2(\Omega_\mathrm{S})^d)} + \|\boldsymbol{\psi} - \pi_h\boldsymbol{\psi}\|_{\mathrm{W}^{2,\infty}((0,T);\mathrm{L}^2(\Sigma)^d)} \\ &\quad + \quad \|\nabla(p - \Pi_h p)\|_{\mathrm{W}^{2,\infty}((0,T);\mathrm{L}^2(\Omega_\mathrm{F})^d)} + (\Delta t)^2\,\|\boldsymbol{\sigma}\|_{\mathrm{W}^{4,\infty}((0,T);\mathrm{H}(\mathbf{div},\Omega_\mathrm{S}))} \\ &\quad + \quad (\Delta t)^2\,\|p\|_{\mathrm{W}^{4,\infty}((0,T);H^1(\Omega_\mathrm{F}))} \Big\}\,,\end{aligned}$$

and the result follows from the norm equivalency provided by Lemma 3.1 and the approximation properties given by (4.1), (4.2) and (4.3)-(4.6). □

6 Numerical results

In this section we present several numerical experiments confirming the good performance of the fully discrete Galerkin scheme (5.1) as applied to a two-dimensional model problem. In all what follows,

$h=\Delta t$	N	$\mathtt{e}_h(\boldsymbol{\sigma})$	$\mathtt{r}_h(\boldsymbol{\sigma})$	$\mathtt{e}_h(\boldsymbol{u})$	$\mathtt{r}_h(\boldsymbol{u})$	$\mathtt{e}_h(p)$	$\mathtt{r}_h(p)$
1/16	7489	8.837e−03	–	8.041e−03	–	9.635e−02	–
1/32	29313	1.929e−03	2.195	1.901e−03	2.081	2.038e−02	2.241
1/64	115969	4.623e−04	2.061	4.688e−04	2.020	4.990e−03	2.030
1/128	461313	1.144e−04	2.014	1.166e−04	2.008	1.257e−03	1.990

Table 6.1: convergence history in the case $\Gamma_{\rm N}=\emptyset$ and $\lambda=\mu=1.0$.

given the solution $(\boldsymbol{\sigma}_h^n, p^n)$ of (5.1) at a time level $n\Delta t$, we postprocess the corresponding displacement field $\boldsymbol{u}_h^n$ by solving the auxiliary saddle point problem:

$$\begin{array}{rcll} \text{Find } \boldsymbol{\sigma}_h^* \in \boldsymbol{\mathcal{W}}_h \text{ with } \boldsymbol{\sigma}_h^*\boldsymbol{n} = -p_h^n\boldsymbol{n} \text{ on } \Sigma,\ \boldsymbol{r}_h^* \in \boldsymbol{\mathcal{Q}}_h \text{ and } \boldsymbol{u}_h^n \in \mathcal{U}_h \text{ such that} & & & \\ (\mathcal{C}^{-1}\boldsymbol{\sigma}_h^* + \boldsymbol{r}_h^*, \boldsymbol{\tau})_{0,\Omega_{\rm S}} + (\boldsymbol{u}_h^n, \mathbf{div}\ \boldsymbol{\tau})_{0,\Omega_{\rm S}} &=& 0 & \forall \boldsymbol{\tau} \in \boldsymbol{\mathcal{W}}_h^{\Sigma}\,, \\ (\mathbf{div}\ \boldsymbol{\sigma}_h^*, \boldsymbol{v})_{0,\Omega_{\rm S}} &=& (\mathbf{div}\ \boldsymbol{\sigma}_h^n, \boldsymbol{v})_{0,\Omega_{\rm S}} & \forall \boldsymbol{v} \in \mathcal{U}_h\,, \\ (\boldsymbol{\sigma}_h^*, \boldsymbol{s})_{0,\Omega_{\rm S}} &=& 0 & \forall \boldsymbol{s} \in \boldsymbol{\mathcal{Q}}_h, \end{array} \tag{6.1}$$

where $\boldsymbol{\mathcal{W}}_h^{\Sigma} := \{\boldsymbol{\tau} \in \boldsymbol{\mathcal{W}}_h; \quad \boldsymbol{\tau}\boldsymbol{n} = \boldsymbol{0}, \quad \text{on } \Sigma\}$.

For each mesh size h, the individual relative errors produced by the fully discrete Galerkin method (5.1) are measured at the final time step as follows:

$$\mathtt{e}_h(\boldsymbol{\sigma}) := \frac{\|\boldsymbol{\sigma}(t_{L-\frac{1}{2}}) - \boldsymbol{\sigma}_h^{L-\frac{1}{2}}\|_{\mathrm{H}(\mathbf{div},\Omega_{\rm S})}}{\|\boldsymbol{\sigma}(t_{L-\frac{1}{2}})\|_{\mathrm{H}(\mathbf{div},\Omega_{\rm S})}}, \quad \mathtt{e}_h(p) := \frac{\|p(t_{L-\frac{1}{2}}) - p_h^{L-\frac{1}{2}}\|_{1,\Omega_{\rm F}}}{\|p(t_{L-\frac{1}{2}})\|_{1,\Omega_{\rm F}}},$$

$$\mathtt{e}_h(\boldsymbol{u}) := \frac{\|\boldsymbol{u}(t_{L-\frac{1}{2}}) - \boldsymbol{u}_h^{L-\frac{1}{2}}\|_{0,\Omega_{\rm S}}}{\|\boldsymbol{u}(t_{L-\frac{1}{2}})\|_{0,\Omega_{\rm S}}},$$

where $\{(\boldsymbol{\sigma}_h^n, p_h^n),\ n=0,\ldots,L\}$ is the solution of (5.1) and $(\boldsymbol{\sigma}, p)$ is the solution of (3.7). In turn, we introduce the experimental rates of convergence

$$\mathtt{r}_h(\boldsymbol{\sigma}) := \frac{\log(\mathtt{e}_h(\boldsymbol{\sigma})/\mathtt{e}_{\hat{h}}(\boldsymbol{\sigma}))}{\log(h/\hat{h})}, \qquad \mathtt{r}_h(p) := \frac{\log(\mathtt{e}_h(p)/\mathtt{e}_{\hat{h}}(p))}{\log(h/\hat{h})},$$

$$\mathtt{r}_h(\boldsymbol{u}) := \frac{\log(\mathtt{e}_h(\boldsymbol{u})/\mathtt{e}_{\hat{h}}(\boldsymbol{u}))}{\log(h/\hat{h})},$$

where $\mathtt{e}_h$ and $\hat{\mathtt{e}}_{\hat{h}}$ are the errors corresponding to two consecutive triangulations with mesh sizes h and $\hat{h}$, respectively.

$h=\Delta t$	N	$\mathtt{e}_h(\boldsymbol{\sigma})$	$\mathtt{r}_h(\boldsymbol{\sigma})$	$\mathtt{e}_h(\boldsymbol{u})$	$\mathtt{r}_h(\boldsymbol{u})$	$\mathtt{e}_h(p)$	$\mathtt{r}_h(p)$
1/16	7489	8.562e−03	–	6.453e−03	–	2.335e−01	–
1/32	29313	1.845e−03	2.214	1.450e−03	2.154	2.657e−02	3.136
1/64	115969	4.412e−04	2.064	3.572e−04	2.021	5.491e−03	2.274
1/128	461313	1.090e−04	2.017	8.905e−05	2.004	1.358e−03	2.016

Table 6.2: convergence history in the case $\Gamma_{\rm N}\neq\emptyset$ and $\lambda=\mu=1.0$.

We now describe the main data of the three examples that will be reported in the following. For each one of them we consider $\Omega_{\rm S}=(0,1)^2\backslash[0.25,0.75]^2$, $\Omega_{\rm F}=(0.25,0.75)^2$, $T=1$, $\rho_S=1$, and $\rho_F=1$.

$h = \Delta t$	N	$\mathtt{e}_h(\boldsymbol{\sigma})$	$\mathtt{r}_h(\boldsymbol{\sigma})$	$\mathtt{e}_h(\boldsymbol{u})$	$\mathtt{r}_h(\boldsymbol{u})$	$\mathtt{e}_h(p)$	$\mathtt{r}_h(p)$
1/16	7489	9.019e−03	−	3.362e−02	−	1.343e+00	−
1/32	29313	1.946e−03	2.212	2.755e−03	3.609	1.808e−01	2.893
1/64	115969	4.673e−04	2.058	8.749e−04	1.655	2.830e−02	2.675
1/128	461313	1.133e−04	2.044	2.404e−04	1.863	6.590e−03	2.103

Table 6.3: convergence history in the case $E = 1.0$, $\nu = 0.49$.

In Example 1, we choose Lamé constants $\lambda = \mu = 1.0$, take $\Gamma_{\mathrm{D}} = \Gamma$ and select the datum $\boldsymbol{f}$ so that the exact solution for the displacement and pressure are given, respectively, by

$$\boldsymbol{u}(\mathbf{x}, t) := \sin(4\pi x_1)\sin(4\pi x_2)\begin{pmatrix}\sin t\\ \sin t\end{pmatrix} \qquad \forall\, \mathbf{x} := (x_1, x_2) \in \Omega_{\mathrm{S}}, \forall\, t > 0\,,$$

and

$$p(\mathbf{x}, t) := \sin(4\pi x_1)\sin(4\pi x_2)\sin(4\sqrt{2}\pi t) \qquad \forall\, \mathbf{x} := (x_1, x_2) \in \Omega_{\mathrm{F}},\ \forall\, t > 0\,.$$

In Example 2, we use again the same displacement and Lamé constants of the first example and choose $\boldsymbol{f}$ so that the exact solution for the pressure is given by

$$p(\mathbf{x}, t) := \sin(x_1 - 0.5)\sin(x_2 - 0.5)\sin(\sqrt{2}t) \qquad \forall\, \mathbf{x} := (x_1, x_2) \in \Omega_{\mathrm{F}},\ \forall\, t > 0\,.$$

In addition, in this case we incorporate the traction boundary condition

$$\boldsymbol{\sigma}\mathbf{n} = \widehat{\boldsymbol{t}} \qquad \text{on} \qquad \Gamma_N\,,$$

with $\Gamma_N := \{x_2 = 0,\ 0 \le x_1 \le 1\}$.

Finally, in Example 3 we test the locking-free character of the method in the nearly incompressible case. For this purpose, we consider now Lamé constants corresponding to a Poisson ratio $\nu = 0.49$ and Young modulus $E = 1.0$, that is

$$\mu = \frac{E}{2(1+\nu)} = 0.336 \quad \text{and} \quad \lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} = 16.443\,,$$

and maintain the displacement, pressure and traction condition of Example 2.

For all the above described examples we consider the AFW elements of order $k = 2$ for the spatial discretization in the solid, and the usual second order Lagrange element for the corresponding discretization in the acoustic medium. Tables 6.1 to 6.3 depict the convergence results obtained by taking equal time and space discretizations parameters Δt and h, respectively. The size of the linear systems solved at each iteration step is indicated by the parameter N. We report on the relative errors and the convergence orders for these three examples. As predicted by the theoretical results, we observe that in all cases the quadratic convergence rate of the error is attained in each variable. In addition, we remark from Example 3 that the method is also robust for nearly incompressible materials, thus confirming its locking-free character.

References

[1] D. N. Arnold, F. Brezzi, and J. Douglas, *PEERS: A new mixed finite element method for plane elasticity*. Japan J. Appl. Math. 1 (1984), no. 2, 347–367.

[2] D. N. Arnold, R. S. Falk, and R. Winther, *Finite element exterior calculus, homological techniques, and applications*. Acta Numer. 15 (2006), 1–155.

[3] D. N. Arnold, R. S. Falk, and R. Winther, *Mixed finite element methods for linear elasticity with weakly imposed symmetry*. Math. Comp. 76 (2007), no. 260, 1699–1723.

[4] D.N. Arnold and J.J. Lee, *Mixed methods for elastodynamics with weak symmetry*. SIAM J. Numer. Anal. 52 (2014), no. 6, 2743–2769.

[5] D. N. Arnold and R. Winther, *Mixed finite elements for elasticity*. Numer. Math. 92 (2002), no. 3, 401–419.

[6] A. Bermúdez, R. Rodríguez, and D. Santamarina, *Finite element approximation of a displacement formulation for time-domain elastoacoustic vibrations*. J. Comput. Appl. Math. 152 (2003), no. 1-2, 17–34.

[7] D. Boffi, F. Brezzi, and M. Fortin, *Reduced symmetry elements in linear elasticity*. Commun. Pure Appl. Anal. 8 (2009), no. 1, 95–121.

[8] D. Boffi, F. Brezzi, and M. Fortin, Mixed Finite Element Methods and Applications. Springer Series in Computational Mathematics, 44. Springer, Heidelberg, 2013.

[9] B. Cockburn, J. Gopalakrishnan, and J. Guzmán, *A new elasticity element made for enforcing weak stress symmetry*. Math. Comp. 79 (2010), no. 271, 1331–1349.

[10] X. Feng, *Analysis of finite element methods and domain decomposition algorithms for a fluid-solid interaction problem*. SIAM J. Numer. Anal. 38 (2000), no. 4, 1312–1336.

[11] C. García, G.N. Gatica, and S. Meddahi, *A new mixed finite element analysis of the elastodynamic equations*. Appl. Math. Lett. 59 (2016), 48–55.

[12] C. García, G.N. Gatica, and S. Meddahi, *A new mixed finite element method for elastodynamics with weak symmetry*. J. Sci. Comput., to appear.

[13] C. García, G.N. Gatica, and S. Meddahi, *Finite element analysis of a pressure-stress formulation for the time-domain fluid-structure interaction problem*. IMA J. Numer. Anal., to appear.

[14] G.N. Gatica, A. Márquez, and S. Meddahi, *Analysis of the coupling of primal and dual-mixed finite element methods for a two-dimensional fluid-solid interaction problem*. SIAM J. Numer. Anal. 45 (2007), no. 5, 2072–2097.

[15] G.N. Gatica, A. Márquez, and S. Meddahi, *Analysis of the coupling of Lagrange and Arnold-Falk-Winther finite elements for a fluid-solid interaction problem in* 3D. SIAM J. Numer. Anal. 50 (2012), no. 3, 1648–1674.

[16] A. Márquez, S. Meddahi, and T. Tran, *Analyses of mixed continuous and discontinuous Galerkin methods for the time harmonic elasticity problem with reduced symmetry*. SIAM J. Sci. Comput., 37 (2015), A1909–A1933.

[17] S. Meddahi, D. Mora, and R. Rodríguez, *Finite element analysis for a pressure-stress formulation of a fluid-structure interaction spectral problem*. Comput. Math. Appl. 68 (2014), no. 12, part A, 1733–1750.

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