UNIVERSIDAD DE CONCEPCIÓN

Centro de Investigación en Ingeniería Matemática (CI^{2}MA)

Centro de Investigación en Ingeniería Matemática

On a finite element approximation for the elastoplastic torsion problem

Franz Chouly, Patrick Hild

PREPRINT 2022-23

SERIE DE PRE-PUBLICACIONES

On a finite element approximation for the elastoplastic torsion problem

Franz Chouly[a,b,c,*], Patrick Hild[d]

[a] *Université Bourgogne Franche-Comté, Institut de Mathématiques de Bourgogne, 21078 Dijon, France*
[b] *Center for Mathematical Modeling and Department of Mathematical Engineering, University of Chile and IRL 2807 CNRS, Santiago, Chile*
[c] *Departamento de Ingeniería Matemática, CI[2]MA, Universidad de Concepción, Casilla 160-C, Concepción, Chile*
[d] *Institut de Mathématiques de Toulouse - UMR CNRS 5219, Université Paul Sabatier, 118 route de Narbonne, 31062 Toulouse Cedex 9, France*

Abstract

This study is concerned with the elastoplastic torsion problem and its standard finite element approximation using piecewise affine Lagrange finite elements. In the case of a polytopal convex domain in dimension $n = 1, 2, 3$ we obtain an H^1-error bound of order h for the solution. For a nonconvex domain, we obtain also an error estimate.

Keywords: variational inequalities; elastoplastic torsion problem; finite elements; error estimates.
2020 MSC: 65N15, 65N30.

1. Introduction

Problems written with weak formulations involving variational inequalities represent various nonlinear phenomena which occur in mechanics and physics [8, 16]. We focus on the elastoplastic torsion problem, as presented in, *e.g.*, [12] (see also [5, 13]). In the aforementioned reference, a direct piecewise affine Lagrange finite element approximation of the variational inequality is also presented, as well as a convergence result (Theorem 3.3), and two error estimates in the H^1-norm, in dimension one (Theorem 3.4) and in dimension two (Theorem 3.5). The error estimate in one dimension is optimal ($\mathcal{O}(h)$), whereas it remained suboptimal in dimension two, as it is of order $\mathcal{O}(h^{\frac{1}{2}-\frac{1}{p}})$ for a source term in L^p, $p > 2$. This bound has not been improved since then, up to our knowledge. Among the few existing results are weak and strong convergence results [19], and error estimates of $\mathcal{O}(h)$ for the L^2-norm of the gradient of the solution and under suitable restrictive assumptions, for mixed finite element approximations, using $\mathbb{P}_1/\mathbb{P}_0$ finite elements [10] or Raviart-Thomas finite elements [6].

In this study we focus on a problem with a positive constant source term. In this case the variational inequality can be reformulated as an "obstacle" problem where the

*Corresponding author
Email addresses: `franz.chouly@u-bourgogne.fr` (Franz Chouly), `patrick.hild@math.univ-toulouse.fr` (Patrick Hild)

May 11, 2022

constraint involves the distance to the boundary, so the obstacle is nonsmooth and the usual techniques from the obstacle problem cannot be directly applied. We present a new direct finite element approximation of the variational inequality, that makes use of piecewise affine, continuous, Lagrange finite elements, and in which the constraint involving the distance function is imposed at each node. When the domain is convex, the discretization is conforming and we prove error estimates in any dimension $n = 1, 2, 3$, with an optimal error bound: $\mathcal{O}(h)$, for a regular enough continuous solution. In the case of a nonconvex domain, an extra term appears due to the nonconformity, that is challenging to bound. We manage to derive an error bound of $\mathcal{O}(h^{3/4})$ for a solution of Sobolev regularity H^α, $\alpha \geq 7/4$.

As usual, we denote by $H^s(.)$, $s \in \mathbb{R}$, the Sobolev spaces. For an open subset D of $\mathbb{R}^n$, the usual norm of $H^s(D)$ is denoted by $\|\cdot\|_{s,D}$. The space $H^1_0(D)$ is the subspace of functions in $H^1(D)$ with vanishing trace on ∂D. The letter C stands for a generic constant, independent of the discretization parameters.

2. The elastoplastic torsion problem

Let $\Omega \subset \mathbb{R}^n$, $n \geq 1$, be an open bounded polytope, connected and with Lipschitz boundary. We consider the variational inequality modelling the torsion of an infinitely long elastoplastic cylinder of cross section Ω and plasticity yield $r > 0$. To simplify we assume that $r = 1$. The problem is to find the stress potential u such that

$$u \in K_1 \ : \qquad a(u, v - u) \geq L(v - u), \qquad \forall\, v \in K_1, \tag{1}$$

where $a : H^1_0(\Omega) \times H^1_0(\Omega) \to \mathbb{R}$ is the bilinear form given by:

$$a(u, v) := \int_\Omega \nabla u \cdot \nabla v, \ \ \forall\, u, v \in H^1_0(\Omega),$$

and $L(v) := \int_\Omega f v, \ \ \forall\, v \in H^1_0(\Omega)$, $f \in L^2(\Omega)$. The notation K_1 represents the nonempty closed convex set of admissible stress potentials:

$$K_1 := \left\{ v \in H^1_0(\Omega) \ : \ |\nabla v| \leq 1 \text{ a.e. in } \Omega \right\},$$

where $|\cdot|$ denotes the euclidian norm in $\mathbb{R}^n$. From Stampacchia's theorem we deduce that Problem (1) admits a unique solution (see also, *e.g.*, [8, 12, 13, 16]).

Remark 2.1. *We recall some regularity results for (1): if $\Omega \subset \mathbb{R}^n$ is open, bounded and convex, with Lipschitz boundary, and for $f \in L^p(\Omega)$ with $n < p < +\infty$, then $u \in W^{2,p}(\Omega) \cap \mathscr{C}^{1,\alpha}(\overline{\Omega})$, where $\alpha = 1 - n/p$ [4]. When the domain is non convex the $W^{2,p}(\Omega)$ regularity can be obtained but the boundary needs to be more regular ($\mathscr{C}^{1,1}$ more precisely, see [11]) so reentrant corners of polytopes are not allowed. When reentrant corners of polytopes are considered, the loss of $W^{2,p}$-regularity is only located near these corners [5].*

Next we suppose that $f = \mathcal{C}$ is a constant function. In this case and according to [3] (see also [15]) the problem (1) can be rewritten as follows: find the stress potential u such that

$$u \in \overline{K} \ : \qquad a(u, v - u) \geq \mathcal{C} \int_\Omega (v - u), \qquad \forall\, v \in \overline{K}, \tag{2}$$

with

$$\overline{K} := \left\{ v \in H_0^1(\Omega) \ : \ |v| \leq d_{\partial\Omega} \text{ a.e. in } \Omega \right\},$$

and $d_{\partial\Omega}$ denotes the (interior) distance function with respect to the boundary $\partial\Omega$:

$$d_{\partial\Omega}(x) := \inf_{y\in\partial\Omega} |x-y|, \ \forall\, x \in \overline{\Omega}.$$

Note that (2) still admits a unique solution from Stampacchia's theorem. To lighten the discussion we can suppose without loss of generality that $\mathcal{C} > 0$, so problem (2) can be rewritten as follows: find the stress potential u such that

$$u \in K \ : \qquad a(u, v-u) \geq \mathcal{C} \int_\Omega (v-u), \qquad \forall\, v \in K, \tag{3}$$

with

$$K := \left\{ v \in H_0^1(\Omega) \ : \ 0 \leq v \leq d_{\partial\Omega} \text{ a.e. in } \Omega \right\}.$$

Again (3) admits a unique solution from Stampacchia's theorem.

Remark 2.2. *We explain why we can consider without loss of generality that $\mathcal{C} > 0$. We see that (2) can be rewritten: find the stress potential u such that*

$$u \in \overline{K} \ : \qquad a(-u, -v-(-u)) \geq -\mathcal{C} \int_\Omega (-v-(-u)), \qquad \forall\, v \in \overline{K}. \tag{4}$$

Since $\overline{K} = -\overline{K}$ and denoting by $u(\mathcal{C})$ the solution of (2) with source term $\mathcal{C}$, we deduce from (4) that $u(-\mathcal{C}) = -u(\mathcal{C})$.

We justify below equivalence between Problem (2) and Problem (3), when $\mathcal{C} > 0$:

Proposition 2.1. *When $\mathcal{C} > 0$, Problem (2) and Problem (3) share the same unique solution $u \in K$.*

Proof. We consider both problems (2) and (3) as minimization problems over their respective sets. Since $K \subset \overline{K}$, these problems are equivalent if the minimizer of $a(\cdot,\cdot)/2 - L(\cdot)$ over $\overline{K}$ lies in fact in K. Let $u \in \overline{K}$ be the solution to (2), and write $u = u_+ - u_-$, with $u_+ = \max(0,u)$ and $u_- = -\min(0,u)$ the positive and negative parts of u, respectively, that both belong to $H_0^1(\Omega)$ [20, Lemma 1.1]. We choose $v = u_+ \in \overline{K}$ in (2) and get:

$$-\|\nabla u_-\|_{0,\Omega}^2 \geq \mathcal{C} \int_\Omega u_-.$$

Since $\mathcal{C} > 0$, we deduce that $u_- = 0$ a.e. in Ω, this means that in fact u lies in K and is also the unique solution to Problem (3). □

Now we consider problem (3) which can be seen as a kind of double obstacle problem.

Remark 2.3. *In the case where $f = \mathcal{C}$ is a positive constant, note that we could write the same variational inequality as in (1),(2) and (3) but with the convex set*

$$\widetilde{K} := \left\{ v \in H_0^1(\Omega) \ : \ v \leq d_{\partial\Omega} \text{ a.e. in } \Omega \right\}.$$

So the torsion problem can be simply seen as an obstacle problem but with a nonsmooth obstacle which is the distance function (roughly speaking the distance function does not lie in $H^2(\Omega)$). This implies that the classical finite element error analysis for the obstacle problem can not be directly applied in the forthcoming analysis.

3. Finite element discretization

Let T_h be a family of simplicial meshes of the domain Ω ($h := \max_{T \in T_h} h_T$ where h_T is the diameter of T). The family of meshes is supposed regular and quasi-uniform. Let V_h be a family of Lagrange finite element spaces of degree one indexed by h, and defined precisely as:

$$V_h := \{v_h \in \mathscr{C}(\overline{\Omega}) \cap H_0^1(\Omega) : \ v_{h|_T} \in \mathbb{P}_1(T), \ \forall \, T \in T_h\}.$$

Let $\mathcal{N}_h$ be the set of the nodes of the mesh and set

$$K_h := \{v_h \in V_h : 0 \leq v_h(a) \leq d_{\partial\Omega}(a), \ \forall a \in \mathcal{N}_h\}.$$

The discrete problem is as follows (recall that $\mathcal{C} > 0$):

$$u_h \in K_h \ : \qquad a(u_h, v_h - u_h) \geq \mathcal{C} \int_\Omega (v_h - u_h) \qquad \forall \, v_h \in K_h, \tag{5}$$

and it admits a unique solution.

Remark 3.1. *If Ω contains a reentrant corner (take for instance a L-shaped domain when $n = 2$), it is easy to check that generally $K_h \not\subset K$. Indeed take $\Omega = (-1,1)^2 \setminus [0,1)^2$, for $n = 2$, and choose a mesh T_h of Ω that contains an edge E between nodes $a_1 = (-0.3-\alpha, -0.3+\alpha)$ and $a_2 = (-0.3+\alpha, -0.3-\alpha)$ with α small enough (so that $(0,0)$ is their closest boundary point). Take $v_h \in K_h$ such that $v_h(a_1) = v_h(a_2) = d_{\partial\Omega}(a_1) = d_{\partial\Omega}(a_2)$. Then, by linear interpolation on E, there holds $v_h(-0.3,-0.3) = d_{\partial\Omega}(a_1) = d_{\partial\Omega}(a_2) > d_{\partial\Omega}((-0.3,-0.3))$ and we deduce that $v_h \notin K$. Note however that, when Ω is a convex set in $\mathbb{R}^n$ there holds $K_h \subset K$, since the hypograph of $d_{\partial\Omega}$ is convex [14, Chapter B, Section 1.3].*

4. *A priori* error estimate

Our main result is:

Theorem 4.1. *Let $\Omega \subset \mathbb{R}^n$, $1 \leq n \leq 3$, be an open bounded polytope, connected and with Lipschitz boundary.*

1. Let Ω be convex, $u \in K \cap H^2(\Omega)$ and $u_h \in K_h$ be the solutions to problems (3) and (5), respectively. There holds

$$\|u - u_h\|_{1,\Omega} \leq Ch\|u\|_{2,\Omega}. \tag{6}$$

2. Let Ω be nonconvex, $u \in K \cap H^\alpha(\Omega)$ ($\max(1, n/2) < \alpha \leq 2$), $\Delta u \in L^2(\Omega)$ and $u_h \in K_h$ be the solutions to problems (3) and (5), respectively. There holds

$$\|u - u_h\|_{1,\Omega} \leq Ch^{\min(3/4, \alpha-1)}\|u\|_{\alpha,\Omega}. \tag{7}$$

Remark 4.1. *1. In the one dimensional case, since Ω is connected, it is necessarily convex, so we recover the well known optimal result of order $\mathcal{O}(h)$ (see,* e.g.*, [13]).*

2. In the convex case, the solution is known to be in $W^{2,p}(\Omega)$ for any $1 < p < \infty$ [4].

3. In the nonconvex case we have to add the assumption $\Delta u \in L^2(\Omega)$ which is necessary to write Falk's lemma in its standard form. Otherwise this would lead to additional technicalities (and changes for the convergence rate of course) which are beyond the scope of this paper. Note that reference [5] investigates some regularity properties of the solution to the torsion problem near reentrant corners.

Proof: From standard Falk's Lemma (see, *e.g.*, [13]), since $\Delta u \in L^2(\Omega)$ and since $\mathcal{I}_h u \in K_h$, where $\mathcal{I}_h$ is the Lagrange interpolation operator mapping onto V_h, we get

$$\begin{aligned}\|u-u_h\|_{1,\Omega}^2 &\le C\Big[\inf_{v_h\in K_h}(\|u-v_h\|_{1,\Omega}^2+\|u-v_h\|_{0,\Omega})+\inf_{v\in K}\|v-u_h\|_{0,\Omega}\Big]\\ &\le C\Big[\|u-\mathcal{I}_h u\|_{1,\Omega}^2+\|u-\mathcal{I}_h u\|_{0,\Omega}+\inf_{v\in K}\|v-u_h\|_{0,\Omega}\Big],\end{aligned} \tag{8}$$

where the constant C depends on $\|\Delta u\|_{L^2(\Omega)}$.

1. In the first case (*i.e.*, Ω is convex and $u \in K \cap H^2(\Omega)$) the first two terms in (8) are bounded by Ch^2 and the second infimum disappears according to Remark 3.1. So bound (6) holds.

2. Let Ω be nonconvex, $u \in K \cap H^\alpha(\Omega)$ with $\max(1, n/2) < \alpha \le 2$ and $\Delta u \in L^2(\Omega)$. From standard approximation bounds, the first two terms in (8) are bounded by $Ch^{2(\alpha-1)}$. To bound the infimum on K, we set $v := \min(u_h, d_{\partial\Omega})$. Clearly $v \in H^1(\Omega)$. Indeed for Ω a bounded polytope there holds $d_{\partial\Omega} \in H^1(\Omega)$, and the minimum of two functions in $H^1(\Omega)$ remains in $H^1(\Omega)$ [20, Lemma 1.1]. Moreover we have $v = 0$ on $\partial\Omega$ and $0 \le v \le d_{\partial\Omega}$, which guarantees that $v \in K$.

Now set $S_h := \{x \in \Omega,\ d_{\partial\Omega}(x) < u_h(x)\}$. This set is generally nonempty since $K_h \not\subset K$. If $x \notin S_h$, then $v(x) = u_h(x)$ by definition. So

$$\|v-u_h\|_{0,\Omega}^2 = \int_\Omega (v-u_h)^2 = \int_{S_h}(d_{\partial\Omega}-u_h)^2.$$

Since $u_h \in K_h$ we have $u_h(a) \le d_{\partial\Omega}(a) = \mathcal{I}_h d_{\partial\Omega}(a), \forall a \in \mathcal{N}_h$. So $\mathcal{I}_h d_{\partial\Omega} - u_h \ge 0$ in Ω. Let $x \in S_h$, then we bound:

$$\begin{aligned}0 < |(u_h-d_{\partial\Omega})(x)| &= (u_h-d_{\partial\Omega})(x)\\ &= (u_h-\mathcal{I}_h d_{\partial\Omega})(x)+(\mathcal{I}_h d_{\partial\Omega}-d_{\partial\Omega})(x)\\ &\le (\mathcal{I}_h d_{\partial\Omega}-d_{\partial\Omega})(x).\end{aligned}$$

Therefore

$$\|v-u_h\|_{0,\Omega}^2 = \int_{S_h}(d_{\partial\Omega}-u_h)^2 \le \int_{S_h}(I_h d_{\partial\Omega}-d_{\partial\Omega})^2 \le \|d_{\partial\Omega}-\mathcal{I}_h d_{\partial\Omega}\|_{0,\Omega}^2.$$

We now consider two different regions of Ω. First, the medial axis which is, for a polytope, the set of its points which have more than one closest point on the boundary (see, *e.g.*, [17, Fig. 4] for an example in two dimensions). On the medial axis, the distance function to the boundary is generally not regular [2, Section 2.3]. The medial axis for a polytope in $\mathbb{R}^n$ is composed of a finite number of $n-1$ dimensional sets (possibly not straight or planar if the polytope is nonconvex) of finite measure in $\mathbb{R}^{n-1}$ (see, *e.g.*, [1, 7, 18]). Since the mesh is regular and quasi-uniform, there are at most C/h^{n-1} simplices T_M intersecting the medial axis. We consider such a simplex T_M, and using the interpolation estimate $\|d_{\partial\Omega}-\mathcal{I}_h d_{\partial\Omega}\|_{L^\infty(T_M)} \le Ch_T\|\nabla d_{\partial\Omega}\|_{L^\infty(T_M)}$ [9, Theorem 1.103], we bound as follows:

$$\|d_{\partial\Omega}-\mathcal{I}_h d_{\partial\Omega}\|_{0,T_M}^2 \le h_T^n\|d_{\partial\Omega}-\mathcal{I}_h d_{\partial\Omega}\|_{L^\infty(T_M)}^2 \le Ch_{T_M}^{n+2}.$$

Since there are at most C/h^{n-1} simplices T_M concerned by the above estimate, we get $\|d_{\partial\Omega} - \mathcal{I}_h d_{\partial\Omega}\|^2_{0,\Omega_M} \leq Ch^3$ where Ω_M stands for the set of simplices intersecting the medial axis.

Consider now the elements T_R which do not intersect the medial axis. In this case the distance function is $C^{1,1}$ (see, *e.g.*, [2, 5, 18]), so its gradient is Lipschitz and almost everywhere differentiable. Then we can bound

$$\|d_{\partial\Omega} - \mathcal{I}_h d_{\partial\Omega}\|_{0,T_R} \leq Ch^2_{T_R} |d_{\partial\Omega}|_{H^2(T_R)}.$$

By summation, we get $\|d_{\partial\Omega} - \mathcal{I}_h d_{\partial\Omega}\|^2_{0,\Omega\setminus\Omega_M} \leq Ch^4$. As a result we obtain

$$\inf_{v\in K} \|v - u_h\|_{0,\Omega} \leq Ch^{3/2}$$

and the final bound

$$\|u - u_h\|_{1,\Omega} \leq Ch^{\min(3/4,\alpha-1)}$$

follows. □

Acknowledgements

The authors thank the two anonymous referees which comments helped to improve the presentation of their results. F.C.s work is partially supported by the I-Site BFC project NAANoD and the EIPHI Graduate School (contract ANR-17-EURE-0002).

References

[1] D. Attali, J.D. Boissonnat, H. Edelsbrunner, Stability and computation of medial axes: a state-of-the-art report, in: Mathematical foundations of scientific visualization, computer graphics, and massive data exploration, Math. Vis., Springer, Berlin, 2009, pp. 109–125.

[2] L. Birbrair, M.P. Denkowski, Medial axis and singularities, J. Geom. Anal. 27 (2017) 2339–2380.

[3] H. Brézis, M. Sibony, Équivalence de deux inéquations variationnelles et applications, Arch. Rational Mech. Anal. 41 (1971) 254–265.

[4] H. Brezis, G. Stampacchia, Sur la régularité de la solution d'inéquations elliptiques, Bulletin de la Société Mathématique de France 96 (1968) 153–180.

[5] L.A. Caffarelli, A. Friedman, The free boundary for elastic-plastic torsion problems, Trans. Amer. Math. Soc. 252 (1979) 65–97.

[6] A. Bermúdez de Castro López, A mixed method for the elastoplastic torsion problem, IMA J. Numer. Anal. 2 (1982) 325–334.

[7] F. Chazal, R. Soufflet, Stability and finiteness properties of medial axis and skeleton, J. Dynam. Control Systems 10 (2004) 149–170.

[8] G. Duvaut, J.L. Lions, Inequalities in mechanics and physics, volume 219 of *Grundlehren der Mathematischen Wissenschaften*, Springer-Verlag, Berlin-New York, 1976.

[9] A. Ern, J.L. Guermond, Theory and practice of finite elements, volume 159 of *Applied Mathematical Sciences*, Springer-Verlag, New York, 2004.

[10] R.S. Falk, B. Mercier, Error estimates for elasto-plastic problems, RAIRO Anal. Numér. 11 (1977) 135–144, 219.

[11] C. Gerhardt, Regularity of solutions of nonlinear variational inequalities with a gradient bound as constraint, Arch. Rational Mech. Anal. 58 (1975) 309–315.

[12] R. Glowinski, Lectures on numerical methods for nonlinear variational problems, volume 65 of *Tata Institute of Fundamental Research Lectures on Mathematics and Physics*, Tata Institute of Fundamental Research, Bombay; Springer-Verlag, Berlin-New York, 1980.

[13] R. Glowinski, J.L. Lions, R. Trémolières, Numerical analysis of variational inequalities, volume 8 of *Studies in Mathematics and its Applications*, North-Holland Publishing Co., Amsterdam-New York, 1981.

[14] J.B. Hiriart-Urruty, C. Lemaréchal, Fundamentals of convex analysis, Grundlehren Text Editions, Springer-Verlag, Berlin, 2001.

[15] G. Idone, A. Maugeri, C. Vitanza, Variational inequalities and the elastic-plastic torsion problem, J. Optim. Theory Appl. 117 (2003) 489–501.

[16] D. Kinderlehrer, G. Stampacchia, An introduction to variational inequalities and their applications, volume 88 of *Pure and Applied Mathematics*, Academic Press, Inc., New York-London, 1980.

[17] D.T. Lee, Medial axis transformation of a planar shape, IEEE Transactions on Pattern Analysis and Machine Intelligence (1982) 363–369.

[18] Q. Mérigot, Détection de structure géométrique dans les nuages de points, Ph.D. thesis, Université Nice Sophia Antipolis, 2009.

[19] K. Mouallif, Approximation du problème de la torsion élasto-plastique d'une barre cylindrique par régularisation et discrétisation d'un problème inf-sup sur $H_0^1(\Omega)\times L_+^\infty(\Omega)$, Travaux Sém. Anal. Convexe 12 (1982) exp. no. 1, 24.

[20] G. Stampacchia, Le problème de Dirichlet pour les équations elliptiques du second ordre à coefficients discontinus, Ann. Inst. Fourier (Grenoble) 15 (1965) 189–258.

Centro de Investigación en Ingeniería Matemática (CI^{2}MA)

PRE-PUBLICACIONES 2022

2022-12 Sergio Caucao, Gabriel N. Gatica, Juan P. Ortega: *A posteriori error analysis of a Banach spaces-based fully mixed FEM for double-diffusive convection in a fluid-saturated porous medium*

2022-13 Raimund Bürger, Julio Careaga, Stefan Diehl, Romel Pineda: *A model of reactive settling of activated sludge: comparison with experimental data*

2022-14 Gabriel N. Gatica, Nicolas Nuñez, Ricardo Ruiz-Baier: *New non-augmented mixed finite element methods for the Navier-Stokes-Brinkman equations using Banach spaces*

2022-15 Dibyendu Adak, David Mora, Alberth Silgado: *A Morley-type virtual element approximation for a wind-driven ocean circulation model on polygonal meshes*

2022-16 Sergio Caucao, Eligio Colmenares, Gabriel N. Gatica, Cristian Inzunza: *A Banach spaces-based fully-mixed finite element method for the stationary chemotaxis-Navier-Stokes problem*

2022-17 Felipe Lepe, David Mora, Gonzalo Rivera, Iván Velásquez: *A posteriori virtual element method for the acoustic vibration problem*

2022-18 Franz Chouly: *A review on some discrete variational techniques for the approximation of essential boundary conditions*

2022-19 Veronica Anaya, Ruben Caraballo, Sergio Caucao, Luis F. Gatica, Ricardo Ruiz-Baier, Ivan Yotov: *A vorticity-based mixed formulation for the unsteady Brinkman-Forchheimer equations*

2022-20 Olga Barrera, Stéphane P. A. Bordas, Raphaël Bulle, Franz Chouly, Jack S. Hale: *An a posteriori error estimator for the spectral fractional power of the Laplacian*

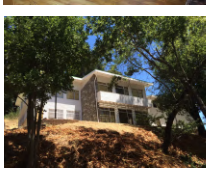
2022-21 Stéphane P. A. Bordas, Raphaël Bulle, Franz Chouly, Jack S. Hale, Alexei Lozinski: *Hierarchical a posteriori error estimation of Bank–Weiser type in the Fenics project*

2022-22 Franz Chouly, Patrick Hild, Vanessa Lleras, Yves Renard: *Nitsche method for contact with Coulomb friction: existence results for the static and dynamic finite element formulations*

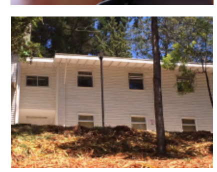
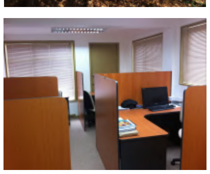
2022-23 Franz Chouly, Patrick Hild: *On a finite element approximation for the elasto-plastic torsion problem*

Para obtener copias de las Pre-Publicaciones, escribir o llamar a: Director, Centro de Investigación en Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile, Tel.: 41-2661324, o bien, visitar la página web del centro: http://www.ci2ma.udec.cl



Centro de Investigación en
Ingeniería Matemática (CI^2MA)
Universidad de Concepción

Casilla 160-C, Concepción, Chile
Tel.: 56-41-2661324/2661554/2661316
http://www.ci2ma.udec.cl