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Problemas de Conjuntos Recubridores de Ciclos y Comportamiento  
Dinámico en Redes Regulatorias  
(Feedback Set Problems and Dynamical Behavior in Regulatory Networks)

*Tesis para optar al grado de  
Doctor en Ciencias Aplicadas con mención en Ingeniería Matemática*

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