Third CI^{2}MA Focus Seminar
"Optimization: Asymptotic Analysis, Strong Duality and Relaxation"

April 17, 2012
Auditorio Alamiro Robledo
Facultad de Ciencias Físicas y Matemáticas
Universidad de Concepción

Organizer: Fabián Flores-Bazán

Preface

Optimization has became one of the most important areas in Applied Mathematics, and where new techniques or a finer analysis of the existing ones have been developed in recent years. Some of these techniques are presented here in different kind of situations.

No doubt the linear complementarity problem is the most significant model-problem in management science, within the framework of linear programming, convex quadratic programmming, and bimatrix games. Asymptotic analysis which consists in describing the behaviour of the phenomenon very far away is employed in the more general semidefinite linear complementarity problem. This is developed in Rubén's talk, where new existence results are established, as well as continuity properties of the solution set. A further study in this line is carried out by J. López and it will be described in his talk. He introduces new linear transformations defined on a Euclidean Jordan algebra.

The asymptotic analysis approach has been employed obviously to derive existence as well as stability and sensitivity analysis in multiobjective programming: this is illustrated in Fernando and Cristián's talks.

Strong duality is a nice property to be studied as a part of duality theory, and became a nice challenge in mathematics due to the lack of convexity. Asymptotic analysis is described in Gabriel's talk providing sufficient and/or necessary conditions for the validity of strong duality as well as existence and optimality conditions in (non convex) quadratic programming; whereas a different approach, based on an image space analysis, is introduced in Giandomenico's talk.

Fabián's talk discusses strong duality in an abstract framework and applies it to a simple problem in the Calculus of Variations, providing also existence of solutions. Asymptotic analysis arises when analysing the epigraph of the optimal value function in terms of extremal points and extremal rays.

Relaxation procedure is part of the asymptotic analysis approach since the former regards the study of limit points of minimizing sequences of the original (minimization)

problem which are minimzers of some relaxed function associated to the original one. Abderrahim's talk shows an instance of this point of view, or a dual version of that.

Fabián Flores-Bazán Autumn, Concepción 2012

Programme

09:55 **Opening**

10:00 **Rubén López** (Universidad Católica de la Santísima Concepción, Chile):
Existence and stability results based on asymptotic analysis for semidefinite linear complementarity problems.

10:45 **Julio López** (Departamento de Matemática, UTFSM):
Characterizing **Q**-Linear transformations for linear complementarity problems. over symmetric cones.

11:30 **Coffee break**

11:45 **Fernando Flores-Bazán** (Universidad del Bío Bío):
Asymptotic methods for generalized vector equilibrium problems problems.

12:30 **Cristián Vera** (Universidad Católica de la Santísima Concepción):
On the set of weakly efficient minimizers for convex multiobjective programming.

13:15 **Lunch**

15:30 **Gabriel Cárcamo** (Departamento de Ingeniería Matemática, Universidad de Concepción):
A geometric characterization of strong duality in nonconvex quadratic programming.

16:15 **Giandomenico Mastroeni** (Dipartimento de Matemática, Università di Pisa):
Nonlinear separation in the image space with applications to lagrangian duality and penalty methods.

17:00 **Coffee Break**

17:15 **Fabián Flores-Bazán** (CI^{2}MA, Departamento de Ingeniería Matemática, Universidad de Concepción):
Existence and strong duality for a class of nonconvex problems in Calculus of Variations.

18:00 **Abderrahim Hantoute** (CMM, UChile):
On the convex relaxation of optimization problems.

18:45 **Closing**

20:30 **Seminar Dinner**

Practical information

Seminar participants who would like to join dinner should register with CI^{2}MA secretary:

Ms Angelina Fritz
CI^{2}MA, office 24
E-mail: afritz@ci2ma.udec.cl
Phone: (041) 266 1324

Abstracts

A geometric characterization of strong duality in nonconvex quadratic programming

Gabriel Cárcamo[1]

We first establish a relaxed version of Dines theorem associated to quadratic minimization problems with finitely many linear equality and a single quadratic inequality constraints. Then, we characterize geometrically the strong duality, and some relationships with the conditions employed in Finsler theorem are established. Our results can be used to situations where none of the results appearing elsewhere are applicable. In addition, a refinement and an improvement of the Frank and Wolfe theorem along with that due to Eaves are proved for asymptotically linear sets.

Keywords. Strong duality, Nonconvex quadratic programming, Relaxed Dines's theorem.

Talk based on joint work with Fabián Flores-Bazán (CI^{2}MA, Departamento de Ingeniería Matemática, Universidad de Concepción).

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[1]Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile.
E-mail: gabrielcaraven@gmail.com

Existence and strong duality for a class of nonconvex problems in Calculus of Variations

Fabián Flores-Bazán[1]

In this paper we start by providing general sufficient conditions that ensure strong duality for a cone constrained nonconvex optimization problem. Our conditions can be used where no previous result is applicable, even in a finite dimensional setting. An application to Calculus of Variations without convexity, yielding existence of solutions and the strong duality property, is provided.

This work has been supported by Proyecto FONDECYT 112-0980 through CONICYT-Chile.

Keywords. Quasi relative interior, strong duality, nonconvex optimization, Liapunov convexity theorem.

Talk based on joint work with Giandomenico Mastroeni (U. di Pisa).

[1]CI^{2}MA, Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile.
E-mail: `fflores@ing-mat.udec.cl`

Asymptotic methods for generalized vector equilibrium problems

Fernando Flores-Bazán[1]

Many problems that appear in applications, mainly in economy, can be modeled mathematically as a vector variational inequality problem, or more generally as a vector equilibrium problem. In this talk I will try to give an overall theoretical framework that enables us to deal with as special cases of vector complementarity and vector optimizations problems. Such a formulation consists of:

$$\text{find } \bar{x} \in K \ \text{ such that } \ F(\bar{x}, y) \notin -\text{int } C \qquad \forall\ y \in K, \tag{E}$$

Where K it is convex and closed subset of a real reflexive Banach space X; $F : K \times K \to Y$ a vector function with Y being a real normed space equipped with a given order by a proper closed, convex, cone C, such that int $C \neq \emptyset$. Problem (E) we will call generalized vector equilibrium problem. Purpose of this talk is establish some sufficient and/or necessary conditions for the solvability to Problem (E). In addition, we also provide some characterizations for the nonemptiness and boudnedness of the solution set to (E).

Keywords. Vector equilibrium problem, asymptotic analysis, weak efficiency.

Talk based on joint work with Q. H. Ansari (Aligarh Muslim University).

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[1]Departamento de Matemática, Universidad del Bío Bío, Avenida Collao 1202, Casilla 5-C, Concepción, Chile.
E-mail: fflores@ubiobio.cl

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On the convex relaxation of optimization problems

Abderrahim Hantoute[1]

We relate a given optimization problem "$\inf_X f$" to its lower semicontinuity convex relaxation "$\inf_X \ (\mathrm{cl}-\mathrm{co})(f)$"; here $(\mathrm{cl}-\mathrm{co})(f)$ is the lower semicontinuity convex hull of f. We establish a complete characterization of the solutions set of the relaxed problem by means exclusively of some kind of the solution of the initial problem. Under some natural conditions, of coercivity type, this analysis yields both existence and characterization of the solution of the initial problem. Our main tool is the subdifferential analysis of the so-called Legendre-Fenchel function.

Keywords. Convex optimization, Convex relaxation, Fenchel conjugate.

[1]CMM, Universidad de Chile, Avenida Blanco Encalada 2120, Santiago, Chile.
E-mail: ahantoute@dim.uchile.cl

Characterizing Q-Linear transformations for linear complementarity problems over symmetric cones

Julio López[1]

In this work, we introduce a new class, called **F**, of linear transformations defined on a Euclidean Jordan algebra. This concept is illustrated in some known examples of Euclidean Jordan algebras: n-dimensional vectors, quadratic forms and n-dimensional symmetric matrices. Also, within this new class, we show the equivalence between **Q**- and $\mathbf{Q_b}$-transformations. We also provide conditions under which a linear transformation belongs to **F**. Finally, we present some examples of transformation: Lyapunov, Quadratic, Stein and relaxation transformation.

Keywords. Euclidean Jordan algebra, linear complementarity problem, $\mathbf{Q_b}$-transformation.

Talk is based on joint work with Rubén López (UCSC, Concepción, Chile) and Héctor Ramírez (CMM-UChile, Santiago, Chile).

[1]Departamento de Matemática, Universidad Técnica Federico Santa María, Vicuña Mackenna 3939, Santiago, Chile.
E-mail: julio.lopez@usm.cl

Existence and stability results based on asymptotic analysis for semidefinite linear complementarity problems

Rubén López[1]

This talk is devoted to the study of existence and stability results of semidefinite linear complementarity problems (for short SDLCP). Our approach consists of approximating the variational inequality formulation of the SDLCP by a sequence of suitable chosen variational inequalities. This provides particular estimates for the asymptotic cone of the solution set of the SDLCP. We thus obtain new coercive and noncoercive existence results, as well as new properties related to the continuity of the solution sets of the SDLCP (such as outer/upper semicontinuity, Lipschitz-type continuity, among others). Moreover, this asymptotic approach leads to a natural extension of the class of García linear transformations, formerly defined in the context of linear complementarity problems, to this SDLCP setting.

This work has been supported by Proyecto FONDECYT 1100919 through CONICYT-Chile.

Keywords. Approximation, variational analysis, optimality.

Talk based on joint work with Julio López (UTFSM, Santiago, Chile) and Héctor Ramírez (CMM-UChile, Santiago, Chile).

[1]Departamento de Matemática y Física Aplicadas, Universidad Católica de la Santísima Concepción, Alonso Ribera 2850, 409-0541 Concepción, Chile.
E-mail: rlopez@ucsc.cl

Nonlinear separation in the image space with applications to lagrangian duality and penalty methods

Giandomenico Mastroeni[1]

The image space analysis is a unifying scheme for studying constrained extremum problems variational inequalities, and, more gener- ally, can be applied to any kind of problem (P) that can be expressed under the form of the impossibility of a parametric system. In this approach, the impossibility of such a system is reduced to the disjunction of two suitable subsets $\mathcal{K}$ and $\mathcal{H}$ of the image space associated with (P). $\mathcal{K}$ is defined by the image of the functions involved in (P), while $\mathcal{H}$ is a convex cone that depends only on the type of conditions (equalities, inequalities, etc) on the class of problems to which (P) belongs. The disjunction of $\mathcal{K}$ and $\mathcal{H}$ can be proved by showing that they lie in two disjoint level sets of a suitable separating functional. In this talk we analyse a nonlinear separation scheme in the image space associated with an infinite dimensional cone constrained extremum problem, and, in particular, we consider the applications to nonlinear lagrangian duality and exact and inexact exterior penalty methods. Duality and exact penalty methods arise from the existence of a regular nonlinear separation, which is shown to be equivalent to a saddle point condition for a generalized Lagrangian function associated with the given problem, while inexact penalty methods are closely related with the existence of an asymptotic nonlinear separation.

Keywords. Image space, nonlinear separation, duality, penalty methods.

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[1]Dipartimento de Matematica, Università di Pisa, L.go B. Pontecorvo 5, 56127, Pisa, Italy.
E-mail: mastroen@dm.unipi.it

On the set of weakly efficient minimizers for convex multiobjective programming

Cristián Vera[1]

In this paper by employing an asymptotic analysis we develop an existence and stability theory for convex multiobjective programming in finite dimensional spaces. To do this we employ a notion of convergence for vector-valued functions close to that due to Lemaire. We obtain stability results of the set of weakly efficient minimizers. We also provide bounds for the solution set and its asymptotic cone.

Keywords. Multiobjective programming, set convergence, horizon limits, epigraphical convergence, asymptotic analysis.

Talk based on joint work with Rubén López (Universidad Católica de la Santísima Concepción).

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[1]Departamento de Matemática y Física Aplicadas, Universidad Católica de la Santísima Concepción, Alonso Ribera 2850, 409-0541 Concepción, Chile.
E-mail: cristianv@ucsc.cl

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