Seminario de Análisis Numérico y Modelación Matemática

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A new vorticity-velocity-pressure fully-mixed formulation for the Brinkman-Darcy problem in 3D

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Resumen

In this work, we propose and analyze a fully-mixed finite element method to numerically approximate the flow patterns of a viscous fluid within a highly permeable medium, described by Brinkman equations, and its interaction with pure porous media flow under Darcy's law. The system is formulated in terms of velocity and pressure of the porous medium, together with vorticity, velocity and pressure of the fluid. In addition, the tangential component of the vorticity is supposed to vanish on the whole boundary of the fluid, whereas null normal components of both velocities are assumed on the respective boundaries, except on the interface where suitable transmission conditions are considered. In this way, the derivation of the corresponding mixed variational formulation leads to a Lagrange multiplier enforcing the pressure continuity across the interface, whereas mass balance results from essential boundary conditions on each domain. As a consequence, a typical saddle-point operator equation is obtained, and hence the classical Babuška-Brezzi theory is applied to establish the well-posedness of the continuous and discrete schemes. In particular, we remark that the continuous and discrete inf-sup conditions of the main bilinear form are proved by using suitably chosen injective operators to get lower bounds of the corresponding suprema, which constitutes a previously known technique, recently denominated T-coercivity. In turn, and consistent with the above, the stability of the Galerkin scheme requires that the $\mathbf{curl}$ of the finite element subspace approximating the vorticity be contained in the space where the discrete velocity of the fluid lives, which yields Raviart-Thomas and Nédélec finite element subspaces as feasible choices. Then we show that the aforementioned constraint can be avoided by augmenting the mixed formulation with a residual arising from the Brinkman momentum equation. Finally, several numerical examples illustrating the good performance of the methods and confirming the theoretical rates of convergence are reported.