Seminario de Análisis Numérico y Modelación Matemática

GIMNAP-Departamento de Matemática, UBB
Centro de Investigación en Ingeniería Matemática (CI^{2}MA), UDEC

Expositor:

Eligio Colmenares

Centro de Investigación en Ingeniería Matemática (CI^{2}MA), UDEC

Título de la Charla:

An Augmented fully-mixed formulation for the stationary Boussinesq problem

Fecha y Hora:
Martes 19 de Mayo de 2015, 15:30 Horas.

Lugar:
Sala Seminario, Facultad de Ciencias
Universidad del Bío-Bío.

Resumen

In this talk, we present the analysis of a new fully-mixed finite element method for the stationary Boussinesq problem. More precisely, we reformulate a previous primal-mixed scheme for the respective model by holding the same modified pseudostress tensor depending on the pressure, and the diffusive and convective terms of the Navier-Stokes equations for the fluid; and in contrast, we now introduce a new auxiliary vector unknown involving the temperature, its gradient and the velocity for the heat equation. As a consequence, a mixed approach is carried out in heat as well as fluid equation, and differently from the previous scheme, no boundary unknowns are needed, which leads to an improvement of the method not only from both the theoretical and computational but also the physical point of view. In fact, the pressure is eliminated and as a result the unknowns are given by the aforementioned auxiliary variables, the velocity and the temperature of the fluid. In turn, further quantities such as the pressure, the stress and vorticity tensors, the velocity gradient of the fluid, and the temperature gradient can be approximated as a simple postprocess from the finite element solutions. In addition, for reasons of suitable regularity conditions, the scheme is augmented by using the constitutive and equilibrium equations, and the Dirichlet boundary conditions. Then, the resulting formulation is rewritten as a fixed point problem and its well-posedness is guaranteed by the classical Banach Theorem combined with the Lax-Milgram Theorem. As for the associated Galerkin scheme, the Brouwer and the Banach fixed point Theorems are utilized to establish existence and uniqueness of discrete solution, respectively. In particular, Raviart-Thomas spaces of order k for the auxiliary unknowns and continuous piecewise polynomials of degree $\leq k+1$ for the velocity and the temperature become feasible choices. Finally, we derive optimal a priori error estimates and provide several numerical results illustrating the good performance of the scheme and confirming the theoretical rates of convergence for all the unknowns as well as the other physical variables.