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A nonlocal upstream-downstream traffic model

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Abstract

In this talk we analyze the well-posedness of a new class of macroscopic vehicular traffic model propoded in [4] which is described by a scalar nonlocal conservation law that simultaneously incorporates both upstream and downstream effects in the flow dynamics. Unlike nonlocal models previously described in the literature, which only account for downstream density averages (look-ahead behavior), the proposed model introduces an additional term depending on an upstream average (look-behind), allowing for a more realistic representation of anticipatory driver behavior under high-density conditions. Inspired by the multiplicative flux proposed in [I. Karafyllis, D. Theodosis, and M. Papageorgiou. Analysis and control of a non-local pde traffic flow model. *International Journal of Control*, 95(3):660–678, 2022], our model generalizes and adapts such ideas to an entropy weak solution framework, allowing for the presence of discontinuities and shock waves. The considered flux takes the form $\rho g(\rho) W(\hat{R}_\delta) V(R_\eta)$, where the nonlocal terms $\hat{R}_\delta$ and R_η represent backward- and forward-looking spatial averages of the density, respectively, and the functions W and V encode the drivers' responses to these observations. The main novelty of this work lies in establishing the existence and uniqueness theory for entropy weak solutions, together with a rigorous proof of Lipschitz continuous dependence of solutions not only on the initial data, but also on the kernel functions, under reasonable structural assumptions on the flux components. The proofs are achieved through the design of a conservative numerical scheme that preserves key structural properties of the continuous model, such as maximum principle, mass conservation, **BV** estimates, and **L**¹-stability. Finally, we present numerical experiments that illustrate the behavior of solutions and the qualitative impact of nonlocal terms on traffic dynamics.

This presentation is based on joint research with Harold Contreras (USS-Chile) and Paola Goatin (INRIA-Sophia-Antipolis, France).

References

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