Jin-Xin's Relaxation Solvers with Defect Measure Corrections

Frédéric Coquel*

March 6, 2015

We present a class of finite volume methods for approximating entropy weak solutions of non-linear hyperbolic PDEs. The main motivation is to resolve discontinuities as well as Glimm's scheme, but without the need for solving Riemann problems exactly. The sharp capture of discontinuities is known to be mandatory in situations where discontinuities are sensitive to viscous perturbations while exact Riemann solutions may not be available (typically in phase transition problems). More generally, sharp capture also prevent discrete shock profiles from exhibiting over and undershoots, which is decisive in a many applications (in combustion for instance). We propose to replace exact Riemann solutions by self-similar solutions conveniently derived from the Xin-Jin's relaxation framework. In the limit of a vanishing relaxation time, the relaxation source term exhibits Dirac measures concentrated on the discontinuities. We show how to handle those so-called defect measures in order to exactly capture propagating shock solutions while achieving computational efficiencies. The lecture will essential focus on the convergence analysis in the scalar setting. A special attention is paid to the consistency of the proposed correction with respect to the entropy condition. We prove the convergence of the method to the unique Kruvkov's solution. This is a joint work with Shi Jin[1], Jian-Guo Liu[2] and Li Wang[3].

*CNRS and Centre de Mathématiques Appliquées, Ecole Polytechnique, Palaiseau, France

[1]Department of Mathematics, and Institute of Natural Sciences, Shanghai Jiao Tong University, Shanghai 20040, China, and Department of Mathematics, University of Wisconsin-Madison, 480 Lincoln Drive, Madison, WI 53706, USA (jin@math.wisc.edu)

[2]Department of Physics and Department of Mathematics, Duke University, Durham, NC 27708, USA (Jian-Guo.Liu@duke.edu)

[3]Department of Mathematics, University of Wisconsin-Madison, 480 Lincoln Drive, Madison, WI 53706, USA (wangli@math.wisc.edu)