APPLIED MATHEMATICS FOR SOME SEPARATION PROCESSES

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Abstract. The five stages of applied mathematics — modelling, analysis of the direct problem, analysis of the inverse problem, construction and analyses of numerical schemes, and dissemination of the results by simulation and validation against real data — will be exemplified for some industrial separation processes driven by gravity. Particles and dissolved substances that are either valuable or harmful are separated from a liquid by sedimentation, flotation or filtration in mineral processing, waste- and drinking-water treatment, and chemical industries.

Results in the applied fields often only describe stationary situations or contain ad hoc numerical methods for dynamic simulations. Some reasons for this insufficiency can be found in the mathematical challenges that arise already with balance-law multi-phase models in one dimension expressed by systems of nonlinear convection-diffusion-reaction PDEs with spatially discontinuous coefficients and terms that make the systems degenerate from parabolic to hyperbolic depending on the solution. The mathematical and numerical analysis of such systems constitute challenges with the aim of having reliable and efficient numerical schemes, but also tools for model calibration and control or optimization of the processes.

The talk focuses on the five applied-mathematics stages for sedimentation of particles in a liquid [**?**, **?**, **?**, **?**, **?**, **?**], whereas the analogous development for column flotation in mineral processing [**?**, **?**, **?**, **?**] is detailed in the minisymposium *Numerical methods for mineral processing, wastewater treatment, and related applications*, where also a new biofilm-growth model with numerical scheme of slow sand filtration is presented.

Keywords: conservation law, discontinuous flux, degenerate parabolic, inverse problem, sedimentation, flotation

Mathematics Subject Classifications (2010): 35K57, 35L65, 35Q35, 65M06

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