ON SKELETAL MUSCLE: MODELING AND SIMULATION

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Abstract. Skeletal muscles exhibit fascinating structural and mechanical properties. Skeletal muscle is composed of cells collectively referred to as fibers, which themselves contain contractile proteins arranged longtitudinally into sarcomeres. These latter respond to signals from the nervous system, and contract; unlike cardiac muscle, skeletal muscles can respond to voluntary control. Muscles react to mechanical forces - they contain connective tissue and fluid, and are linked via tendons to the skeletal system - but they also are capable of activation via stimulation (and hence, contraction) of the sacromeres. The restorative along-fibre force introduce strong mechanical anisotropy, and depend on departures from a characteristic length of the sarcomeres; diseases such as cerebral palsy cause this characteristic length to change, thereby impacting muscle force.

In the 1910s, A.V. Hill [1] observed muscles heat when they contract, but not when they relax. Based on experiments on frogs he posited a mathematical description of skeletal muscles which approximated muscle as a 1-dimensional nonlinear and massless spring. This has been a remarkably successful model, and remains in wide use. Yet skeletal muscle is three dimensional, has mass, and a fairly complicated structure. Are these features important? What insights are gained if we include some of this complexity in our models? Many mathematical questions of interest in skeletal muscle mechanics arise: how to model this system, how to discretize it, and what theoretical properties does it have?

In this talk, we survey recent work on the modeling, simulation and validation of a fully 3-D continuum elasticity approach for skeletal muscle dynamics. Skeletal muscle is modelled as a fibre- reinforced nonlinear elastic material, with other connective tissues such as aponeurosis and tendon being similarly described. These fibres are capable of nonlinear activation. We use a three-field formulation [2, 6] After discretization (semi-implicit in time, FEM in space), the model is validated against physiological data, and then used to understand the impact of muscle architecture, mass and tissue properties on questions of physiological interest [4, 5].

This is joint work based on a long-standing collaboration with James Wakeling (Dept. of Biomedical Physiology and Kinesiology, SFU).

Keywords: Nonlinear elasticity, skeletal muscle mechanics, three-field formulation.

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