

# MATHEMATICAL ULTRASONICS: ANALYSIS AND SIMULATION

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ABSTRACT. Research in ultrasonics is fueled by a growing number of high-intensity ultrasound applications in medicine and industry. Sound evolution is quasilinear at high intensities or frequencies, and in tissue-like media, nonlocal effects of time-fractional type may come into play. This application field gives rise to many interesting mathematical questions involving such nonlinear and nonlocal wave equations, including discretizations under realistic (smallness) assumptions on the data and dealing with singular behavior in the vanishing limit of medium parameters. In this talk, I will give an overview of these questions and then present recent work on the robust mathematical and numerical analysis of singularly perturbed nonlinear acoustic wave equations. In particular, acoustic equations of Westervelt and Kuznetsov type with (non)local dissipation will be discussed. Here, one can draw parallels from their uniform treatment in the continuous setting, as both rest upon devising appropriate energy functionals that remain stable in the zero parameter limit.

The talk is based on joint works [?]-[?], and [?].

**Keywords:** mathematical ultrasonics, asymptotic-preserving methods

**Mathematics Subject Classifications (2010):** 65M12, 65M15, 35L72, 65M60

## REFERENCES

- [1] B. Dörich and V. Nikolić, Robust fully discrete error bounds for the Kuznetsov equation in the inviscid limit, *in preparation*.
- [2] B. Kaltenbacher and V. Nikolić, Parabolic approximation of quasilinear wave equations with applications in nonlinear acoustics. *SIAM Journal of Mathematical Analysis*, 54(2): 1593–1622, 2022.
- [3] Mostafa Meliani and V. Nikolić, Mixed approximation of nonlinear acoustic equations: Well-posedness and a priori error analysis. *Applied Numerical Mathematics*, to appear.
- [4] V. Nikolić and B. Wohlmuth, A priori error estimates for the finite element approximation of Westervelt’s quasi-linear acoustic wave equation. *SIAM Journal on Numerical Analysis*, 57(4):1897–1918, 2019.
- [5] V. Nikolić, Asymptotic-preserving finite element analysis of Westervelt-type wave equations. *arXiv:2303.10743*, 2023.

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