NUMERICAL ANALYSIS OF A POROUS NATURAL CONVECTION SYSTEM WITH VORTICITY AND VISCOUS DISSIPATION

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Abstract. We propose and analyse a new formulation and pointwise divergence-free mixed finite element methods for the numerical approximation of Darcy–Brinkman equations in vorticity–velocity–pressure form, coupled with a transport equation for thermal energy with viscous dissipative effects and mixed Navier-type boundary conditions. The solvability analysis of the continuous and discrete problems hinges on Banach spaces needed to properly control the advective and dissipative terms in the non-isothermal energy balance equation. Error estimates in appropriate norms are derived, and a few representative numerical examples are provided.

Keywords: Flow–transport coupling; Highly permeable porous media; Vorticity-based formulation; Mixed finite element methods; Analysis in Banach spaces; Viscous dissipation.

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References

[1] M. Amara, E. Chacón Vera, and D. Trujillo. Vorticity–velocity–pressure formulation for Stokes problem. *Math. Comp.*, **73**(248) (2004) 1673–1697.

[2] C. Amrouche and I. Boussetouan. Vector potentials with mixed boundary conditions. Application to the Stokes problem with pressure and Navier-type boundary conditions. *SIAM J. Math. Anal.*, **53**(2) (2021) 1745–1784.

[3] V. Anaya, M. Bendahmane, D. Mora, and R. Ruiz-Baier, On a vorticity-based formulation for reaction-diffusion-Brinkman systems. *Netw. Heterog. Media*, **13**(1) (2018) 69–94.

[4] C.I. Correa and G.N. Gatica. On the continuous and discrete well-posedness of perturbed saddle-point formulations in Banach spaces. *Comput. Math. Appl.*, **117** (2022) 14–23.

[5] V. Maz'ya and J. Rossmann. Lp estimates of solutions to mixed boundary value problems for the Stokes system in polyhedral domains. *Math. Nachr.*, **280**(7) 751–793.

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