MIXED FINITE ELEMENT METHODS FOR FLUID-POROELASTIC STRUCTURE INTERACTION

SERGIO CAUCAO, TONGTONG LI, AND IVAN YOTOV

ABSTRACT. The Stokes-Biot model describes the interaction between a free fluid and a fluid in a poroelastic material. The two regions are coupled via suitable interface conditions, including balance of forces, continuity of normal velocity, and no-slip or slip with friction tangential velocity condition. We develop a fully mixed formulation for the Stokes-Biot model. It is based on weakly symmetric deviatoric stress, velocity, and vorticity for Stokes, weakly symmetric stress, displacement, and rotation for elasticity, and mixed velocity and pressure for Darcy flow. This formulation exhibits multiple advantages, including local conservation of mass for the Darcy fluid, local poroelastic and Stokes momentum conservation, accurate approximations with continuous normal components for the Darcy velocity, the poroelastic stress, and the free fluid stress, locking-free behavior, and robustness with respect to the physical parameters. Well posedness of the variational formulation and its mixed finite element approximation is established. Stability and error analysis is performed for the numerical method. Furthermore, we employ a multipoint stress mixed finite element method for the discretization of the Stokes and elasticity equations, as well as a multipoint flux mixed finite element method for the Darcy flow. The method allows for local elimination of the stresses, Darcy velocity, vorticity, and rotation, resulting in a symmetric and positive definite cell centered system involving only the Stokes velocity, displacement, and Darcy pressure. Numerical results are presented to illustrate the performance of the method, including its flexibility and robustness for several geoscience applications.

Keywords: fluid-poroelastic structure interaction, Stokes-Biot model, mixed finite element methods, multipoint flux methods

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UNIVERSIDAD CATÓLICA DE LA SANTÍSIMA CONCEPCIÓN
Email address: scaucao@ucsc.cl

DARTMOUTH COLLEGE
Email address: tongtong.li@dartmouth.edu

UNIVERSITY OF PITTSBURGH
Email address: yotov@math.pitt.edu