EXISTENCE AND UNIQUENESS OF SOLUTION FOR A FAMILY OF NONLINEAR DEGENERATE MIXED PARABOLIC EQUATIONS AND ITS APPLICATIONS TO EDDY CURRENT MODELS

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ABSTRACT. The aim of this talk is to show an abstract framework to analyze the solvability of a family of nonlinear degenerate mixed parabolic equations. More precisely, we joint the well-known Babuska-Brezzi theory for stationary mixed problems (see, for instance, [8, Section 1.5]) and the theory for nonlinear degenerate parabolic equations (see [10, Section III.6]), taking also inspiration from the hypothesis satisfied by the nonlinear operator in the three fields abstract framework presented by Gatica in [9, Section 2], to obtain sufficient conditions to guarantee the existence and uniqueness of solution.

Furthermore, we illustrate some applications of the abstract setting through particular problems that arise from a physical model that arises from the electromagnetism: nonlinear eddy current models in a general tridimensional bounded domain including conductors and dielectrics (see, for instance, [1, 2, 4]), and the case of an axisymmetrical domain (see [5, 6]).

We obtain in this work a generalization of the abstract framework proposed in 1985 by Bernardi & Raugel [7], since the problem proposed by the two authors considered a non-degenerate mixed linear parabolic equations and our abstract framework, on the other hand, is degenerate and nonlinear (see also [3]). Moreover, we show that the solutions of our problems have a desired regularity if we wish to approximate the solution by using, for instance, the finite element method.

Keywords: Well-posedness, parabolic degenerate equations, mixed problems, nonlinear problems, time-dependent Stokes problem, eddy current model.

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