AN ADAPTIVE SAMPLING STRATEGY TO APPROXIMATE PARTIAL DIFFERENTIAL EQUATIONS FROM NOISY DATA

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Abstract. Many problems in computational science, engineering, and uncertainty quantification (UQ) require the approximation of Partial Differential Equations (PDEs) from corrupted data, which is similar to approximating a high-dimensional function from noisy samples. In many applications, the data is costly to generate: for example, each sample may require a finite element method to solve PDEs. Therefore, it is imperative to develop highly sample-efficient models. Many methods are based on the fidelity data, high-fidelity models output data with high accuracy but it is expensive to compute. On the other hand, low-fidelity models output less accurate data but it is cheap to generate. Multi-fidelity models, which combine a high-fidelity model with several low-fidelity models, have shown accurate predictions using machine learning techniques. However, these proposed methods work with Monte Carlo sampling (MCS) or variations of MCS. Recently, Christoffel Sampling for Machine Learning (CS4ML) has shown accurate results approximating functions on arbitrary types of data, such as approximating the solution of a PDE using Physic-Informed Neural Networks, that consider the PDE, boundary, and initial condition in the loss function of the neural network. In this work, we propose an adaptive sampling strategy for Multi-fidelity UQ models. In particular, it integrates the multi-fidelity models and the sampling framework CS4ML to increase the sample efficiency of the model approximating the PDEs. Our novel approach is based on the Christoffel function on each latent space of the low-fidelity models to construct low-cost sample measures that will improve the approximation of the multi-fidelity model. We test our method on several problems related to PDEs such as 'Thermally-driven cavity fluid flow', which models the temperature-driven fluid flow in a cavity, and 'Composite Beam', which models the deformation of plane stress, and 'Burgers equation'. Our results demonstrate that our method often yields substantial savings in the number of samples required to achieve a given accuracy. These results therefore are a promising step towards fully adapting Multi-fidelity methods towards scientific computing applications.

Keywords: high-dimensional approximation, sampling strategies, Christoffel function, physic-informed neural networks, Burgers equation, finite element methods, uncertainty quantification.

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