CHARACTERIZATION OF SINGULAR FLOWS OF ZEROTH-ORDER PSEUDO-DIFFERENTIAL OPERATORS VIA ELLIPTIC EIGENFUNCTIONS

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Abstract. The propagation of internal gravity waves in stratified media (such as those found in ocean basins and lakes) leads to the development of geometrical patterns called "attractors". These structures accumulate much of the wave energy and make the fluid flow highly singular. In analytical terms, recent results have confirmed the long-time-believed fact that these attractors develop because of the presence of a continuous spectrum in a class of zeroth-order pseudo-differential operators.

In this talk, we discuss recent progress in the study of this phenomenon from a numerical perspective. First, we propose a high-order pseudo-spectral method to solve the evolution problem, whose long-term behavior is known to be non-square-integrable. Then, we use similar tools to discretize the corresponding eigenvalue problem. Since the eigenvalues are embedded in a continuous spectrum, their computation is based on viscous approximations. Finally, we explore the effect that the embedded eigenmodes have on the long-term evolution of the system.

Keywords: spectral methods, pseudo-differential operators, singular solutions, embedded eigenvalues, internal wave attractors.

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