A FIVE-FIELD MIXED FORMULATION FOR STATIONARY MAGNETOHYDRODYNAMIC FLOWS IN PORORUS MEDIA

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ABSTRACT. We introduce and analyze a new mixed variational formulation for a stationary magnetohydrodynamic flows in porous media problem, whose governing equations are given by the steady Brinkman–Forchheimer equations coupled with the Maxwell equations. Besides the velocity, magnetic field and a Lagrange multiplier asssociated to the divergence-free condition of the magnetic field, a convenient translation of the velocity gradient and the pseudostress tensor are introduced as further unknowns. As a consequence, we obtain a five-field Banach spaces-based mixed variational formulation, where the aforementioned variables are the main unknowns of the system. The resulting mixed scheme is then written equivalently as a fixed-point equation, so that the well-known Banach theorem, combined with classical results on nonlinear monotone operators and a sufficiently small data assumption, are applied to prove the unique solvability of the continuous and discrete systems. In particular, the analysis of the discrete scheme requires a quasi-uniformity assumption on mesh. The finite element discretization involves Raviart–Thomas elements of order $k \geq 0$ for the pseudostress tensor, discontinuous piecewise polynomial elements of degree k for the velocity and the translation of the velocity gradient, Nédélec elements of degree k for the magnetic field and Lagrange elements of degree $k+1$ for the associated Lagrange multiplier. Stability, convergence, and optimal *a priori* error estimates for the associated Galerkin scheme are obtained. Numerical tests illustrate the theoretical results.

Keywords: Brinkman–Forchheimer equations; Maxwell equations; Mixed finite element methods; Fixed point theory; A priori error analysis

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REFERENCES

[1] L. ANGELO, J. CAMAÑO, AND S. CAUCAO, *A five-field mixed formulation for stationary magnetohydrodynamic flows in porous media.* Computer Methods in Applied Mechanics and Engineering, vol. 414, Art. Num. 116158, (2023).

[2] Y. AMIRAT, L. CHUPIN, AND R. TOUZANI, *Weak solutions to the equations of stationary magnetohydrodynamic flows in porous media.* Commun. Pure Appl. Anal. 13 (2014), no. 6, 2445–2464.

[3] S. CAUCAO, G.N. GATICA, AND J.P. ORTEGA, *A fully-mixed formulation in Banach spaces for the coupling of the steady Brinkman–Forchheimer and double-diffusion equations.* ESAIM Math. Model. Numer. Anal. 55 (2021), no. 6, 2725–2758.

[4] D. SCHÖTZAU, *Mixed finite element methods for stationary incompressible magneto-hydrodynamics.* Numer. Math. 96 (2004), no. 4, 771–800.

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