NUMERICAL APPROXIMATION OF A FLUID FLOW IN A DEFORMABLE TUBE WITH SLIP BOUNDARY CONDITION OF FRICTION TYPE ON THE INTERFACE

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Abstract. We are interested in the numerical resolution of an unsteady fluid–structure interaction problem with a slip boundary condition. In this problem, an incompressible Newtonian viscous fluid flows in a deformable tube. The fluid and structure behavior are governed by the incompressible Navier–Stokes equation and a generalized membrane equation (see [**?**]), respectively. On the fluid–structure interface, instead of the usual no-slip boundary condition for the fluid, we impose a slip boundary condition of friction type (see [**?**]). This condition allows the fluid to slip on the interface if the fluid shear stress reaches a threshold value. The threshold value could be either a known function or it could depend on the fluid state. The problem is completed with an impermeability condition and with a stress continuity condition (based on the action-reaction principle) between the fluid and the solid.

First, we show that the energy system presents an additional dissipative term due to the slip boundary condition. Next, we present a numerical scheme for the fluid problem based on the Characteristic-ALE method (see, e.g., [**?**]) for the time discretization and on a finite element four-field mixed variational formulation for the space discretization. In this formulation two Lagrange multipliers are introduced in order to release the impermeability condition and the slip boundary condition (see [**?**]). The structure problem is discretized using a Newmark scheme in time and a finite element approximation in space. Finally, the implicit coupling conditions between the fluid and structure, at each time step, are ensured using an iterative scheme. After that, we study the discrete energy balance for the semi-discretized variational problem. Finally, we will present some numerical simulations.

Keywords: Fluid-structure interaction problem, Friction type slip boundary condition, Mixed variational formulation.

Mathematics Subject Classifications (2010): 65M50, 65N30, 76M10, 74F10, 76D07.

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