ADAPTIVE DEEP FOURIER RESIDUAL METHOD FOR SOLVING PDES ON POLYGONAL DOMAINS

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ABSTRACT. The Deep Fourier Residual (DFR) method is a specific case of Variational Physics-Informed Neural Networks (VPINN) [1] among a large range of strategies for solving PDEs using Neural Networks (NNs). The loss function in the DFR method is an approximation of the dual norm of the PDE's weak-residual, that is, $\mathcal{R} : H \to H^*$, where H is a Hilbert space of test functions and H^* its dual. For many well-posed problems, the dual norm of the weak-residual corresponds directly to the energy norm of the error, i.e., there exist constants $0 < \gamma < M$, such that

$$\frac{1}{M}\|\mathcal{R}(u)\|_{H^*} \leq \|u - u^*\|_H \leq \frac{1}{\gamma}\|\mathcal{R}(u)\|_{H^*},$$

where u is an approximation to the solution u^*. Therefore, this loss function ensures that reducing the loss during the training of a NN corresponds to reducing the error in the solution at the same rate.

In [2, 3], the DFR method was proposed for solving problems in $H_0^1(\Omega)$ and $H(\mathrm{curl}, \Omega)$. There, the calculation of the dual norm is based on a spectral representation of the dual norms of the test function space. This spectral representation is well-known on rectangles in 2D or rectangular cuboids in 3D, but constructing an appropriate orthonormal basis in more general domains is non-trivial.

This talk discusses an extension of the DFR method to the use of adaptive strategies on general polygonal domains. We decompose the PDE domain Ω into rectangular subdomains and the loss function is computed as the sum of local loss functions. We then employ a Döfler marking algorithm to adaptatively refine the initial subdomain decomposition of Ω and increase the accuracy of the approximated solution on relevant regions of the domain.

Our numerical results show the generation of quasi-optimal refined meshes on several 1D and 2D problems, including the singular L-shape problem.

Keywords: Deep Learning, Residual minimisation, Physics-informed machine learning, Adaptive numerical methods

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