THE DEEP FOURIER RESIDUAL METHOD FOR PDES: H^1 AND $H(\mathrm{curl})$ TEST SPACES

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Abstract. Solving PDEs with machine learning techniques has become a popular alternative to conventional methods. In this context, Neural networks (NNs) are among the most commonly used machine learning tools, and in those models, the choice of an appropriate loss function is critical. In general, the main goal is to guarantee that minimizing the loss during training translates to minimizing the error in the solution at the same rate. In multiple problems, this error norm coincides with the dual-norm of the residual; however, it is often difficult to accurately compute it. This work assumes rectangular domains and proposes the use of a Discrete Sine/Cosine Transform to accurately and efficiently compute those dual-norms. The resulting Deep Fourier-based Residual (DFR) method efficiently and accurately approximate solutions to PDEs. This is particularly useful when solutions lack from regularity and methods involving strong formulations of the PDE fail. We focus on the case of variational formulations having H^1 and $H(\mathrm{curl})$ test spaces.

Keywords: Deep learning; Neural Networks; Numerical PDEs; Fourier methods.

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