GOAL-ORIENTED ADAPTIVE MULTILEVEL QUASI-MONTE CARLO FOR ELLIPTIC RANDOM PDES

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Abstract. We present our work titled "Goal-oriented adaptive multilevel quasi-Monte Carlo (MLQMC) for elliptic random PDEs," building upon [Beck, Joakim, et al., "Goal-oriented adaptive finite element multilevel Monte Carlo with convergence rates." Computer Methods in Applied Mechanics and Engineering (2022)] and other ongoing research. Our objective is to solve an elliptic partial differential equation (PDE) with lognormal random input data, when the PDE model faces geometry-induced singularity.

Earlier research [Moon, K-S., et al. "Convergence rates for an adaptive dual weighted residual finite element algorithm." BIT Numerical Mathematics 46.2 (2006)] established convergence rates for a goal-oriented adaptive algorithm. This algorithm utilized isoparametric d-linear quadrilateral finite element approximations and the dual weighted residual error representation in a deterministic context. Notably, this algorithm refines the mesh based on the error's impact on the Quantity of Interest (QoI).

Our current work seeks to merge MLMC/MLQMC with the adaptive finite element solver. Unlike traditional Multilevel Monte Carlo methods, where each sample is determined using a discretization-based numerical method (with resolution tied to the level), our adaptive MLMC (AMLMC) algorithm employs a series of tolerances as its levels. Specifically, for a particular realization of the input coefficient and a set accuracy level, the AMLMC formulates its approximate sample using the initial mesh from the sequence of deterministic, non-uniform meshes. These meshes are produced by the previously mentioned adaptive algorithm and meet the sample-dependent bias constraint. Additionally, the incorporation of QMC enhances the convergence rate.

Keywords: Multilevel quasi-Monte Carlo, Goal-oriented adaptivity, Computational complexity, Finite elements, Partial differential equations with random data, Lognormal diffusion

Mathematics Subject Classifications (2010): 65C05 65N50 65N22 35R60

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