A PARALLEL SOLVER FOR FLUID-STRUCTURE INTERACTION PROBLEMS WITH LAGRANGE MULTIPLIER

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Abstract. We consider a fictitious domain formulation with distributed Lagrange multiplier for fluid-structure interaction problems [1]. The evolution of the structure is modeled by Lagrangian description on a reference domain, which is mapped, at each time step, to the actual position of the solid body. The fluid is described by an Eulerian model and its mesh is extended also in the region occupied by the structure: the coupling is weakly enforced making use of a Lagrange multiplier.

We focus on the analysis of parallel block preconditioners for the linear system arising from the finite element discretization of this family of problems [2, 3].

The fluid is governed by the time dependent Stokes equations with velocity and pressure discretized by the popular $\mathcal{Q}_2 - \mathcal{P}_1$ element, while the solid variables are approximated by $\mathcal{Q}_1$ finite elements. For the structure material both linear and nonlinear constitutive laws are considered.

A first order semi-implicit finite difference scheme is considered for the time discretization. At each time step, the linear system is solved by parallel GMRES accelerated by coupled block diagonal or triangular preconditioners; the diagonal blocks are inverted exactly by parallel direct methods. The implementation is based on the PETSc library [4] and several numerical tests have been performed on Linux clusters to investigate optimality and scalability of the proposed solver.

Keywords: fluid-structure interactions, fictitious domain, preconditioners, parallel solver

Mathematics Subject Classifications (2010): 65N30, 65N12, 74F10, 65F08.

References

[1] D. Boffi, N. Cavallini, L. Gastaldi (2015). *The finite element immersed boundary method with distributed Lagrange multiplier*. SIAM J. Numer. Anal. 53 (2015), no. 6, 2584–2604.

[2] D. Boffi, F. Credali, L. Gastaldi, S. Scacchi (2022). *A parallel solver for fluid structure interaction problems with Lagrange multiplier*. arXiv preprint arXiv:2212.13410.

[3] D. Boffi, F. Credali, L. Gastaldi, S. Scacchi (2023). *A parallel solver for FSI problems with fictitious domain approach*. Math. Comput. Appl. 2023, 28, 59.

[4] S. Balay, S. Abhyankar, M. F. Adams, J. Brown, P. Brune, K. Buschelman, L. Dalcin, V. Eijkhout, W. D. Gropp, D. Kaushik, M. G. Knepley, D. A. May, L. C. McInnes, R. T. Mills, T. Munson, K. Rupp, P. Sanan, B. F. Smith, S. Zampini, H. Zhang, and H. Zhang (2018). *PETSc users manual*. Technical Report ANL-95/11 - Revision 3.9, Argonne National Laboratory.

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