A GEOMETRICALLY AND THERMODYNAMICALLY COMPATIBLE FINITE VOLUME SCHEME FOR CONTINUUM MECHANICS ON UNSTRUCTURED VORONOI MESHES

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Abstract. We consider a first order hyperbolic formulation of the equations of continuum mechanics [1], that includes fluid mechanics as well as solid mechanics. This system falls into the larger class of overdetermined hyperbolic and thermodynamically compatible (HTC) systems of partial differential equations. They satisfy an entropy inequality (second principle of thermodynamics) and conserve total energy (first principle of thermodynamics). Furthermore, the governing equations involve a geometric constraint that links the determinant of deformation gradient to the density of the material under study. This is also known as the Geometric Conservation Law (GCL), which can be regarded as an extra conservation law admitted by the governing equations. The aim of this talk is to present a novel geometrically and thermodynamically compatible cell-centered finite volume scheme on unstructured Voronoi meshes. First, the governing equations are written in fluctuation form. Next, the non-compatible numerical fluxes of the deformation gradient are corrected using a scalar correction factor that is defined at the faces of the grid. This allows to achieve full compatibility with the GCL, hence ensuring positivity of density. Eventually, another scalar factor takes into account thermodynamic compatibility by a correction which only acts on the momentum and the total energy equation, in order not to destroy the geometric compatibility previously obtained. This exhibits a direct link with thermodynamic compatibility in Lagrangian coordinates [2]. Entropy preservation can then be attained on general unstructured meshes. The novel schemes are provably compatible at the semi-discrete level and their accuracy and robustness are validated against a set of benchmarks for fluid and solid mechanics.

Keywords: thermodynamically compatible finite volume schemes, Geometric Conservation Law, cell entropy inequality, positivity preserving, unstructured mesh

Mathematics Subject Classifications (2010): please, name your files after the lastname of the first author, that is `lastname.tex`, and send both the .tex and .pdf files.

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