A UNIFIED AND CONSTRUCTIVE FRAMEWORK FOR THE UNIVERSALITY OF NEURAL NETWORKS

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Abstract. One of the reasons why many neural networks are capable of replicating complicated tasks or functions is their universal approximation property. Though the past few decades have seen tremendous advances in theories of neural networks, a single constructive and elementary framework for neural network universality remains unavailable. This paper is an effort to provide a unified and constructive framework for the universality of a large class of activation functions including most of the existing ones. At the heart of the framework is the concept of neural network approximate identity (nAI). The main result is: *any nAI activation function is universal in the space of continuous functions on compacta*. It turns out that most of the existing activation functions are nAI, and thus universal. The framework induces **several advantages** over the contemporary counterparts. First, it is constructive with elementary means from functional analysis, probability theory, and numerical analysis. Second, it is one of the first unified and constructive attempts that is valid for most of the existing activation functions. Third, it provides new proofs for most activation functions. Fourth, for a given activation and error tolerance, the framework provides precisely the architecture of the corresponding one-hidden neural network with a predetermined number of neurons and the values of weights/biases. Fifth, the framework allows us to abstractly present the first universal approximation with a favorable non-asymptotic rate. Sixth, our framework also provides insights into the developments, and hence providing constructive derivations, of some of the existing approaches.

Keywords: Universal approximation; neural networks; activation functions; non-asymptotic analysis

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