

MATHEMATICAL AND NUMERICAL ANALYSIS OF CONSERVATION LAWS WITH CONTINUOUS AND DISCONTINUOUS NONLOCAL FLUXES.

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ABSTRACT. In this talk, we introduce two scalar conservation laws with continuous and discontinuous nonlocal fluxes, which are motivated by applications in sedimentation and vehicular traffic. We approximate the problems through an Hilliges-Weidlich (HW)-type numerical scheme and provide compactness estimates for the sequence of approximate solutions. Then, we prove the existence and the uniqueness of entropy weak solutions. Some numerical simulations illustrate the behaviour of solutions in sample cases and corroborate the theoretical results.

Keywords: Conservation law, nonlocal flux, continuous flux, discontinuous flux, Hilliges-Weidlich type scheme, entropy weak solution.

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