FRONT TRACKING AND PARAMETER IDENTIFICATION FOR A CONSERVATION LAW WITH A SPACE-DEPENDENT COEFFICIENT MODELING GRANULAR SEGREGATION

RAIMUND BÜRGER, YESENNIA MARTÍNEZ, AND LUIS MIGUEL VILLADA

Abstract. A well-known experimental setup for the study of segregation by size in a dry granular medium consists of two layers of spheres composed of large and small rigid spheres within an annular region of concentric cylinders covered above and below by plates. As one of the cylinders is rotated and thereby applies shear to the granular mixture, the spheres mix and the large spheres rise while the small ones settle in vertical direction. An established model for this phenomenon of segregation by size [**?**] can be written as a conservation law whose flux involves a piecewise constant or smooth coefficient that describes dependence of the shear rate on depth. This model can be solved by the hyperfast front tracking method suitably adapted to handle a conservation law with discontinuous flux. It is demonstrated how the problem of identification of the mentioned coefficient from experimental observations can be solved efficiently by the front tracking method. The aim of this paper is to show the efficiency of the Front Trancking method against a numerical scheme, in our case we chose Engquist-Osher, for the identification of parameters in an inverse problem that models the phenomenon of granular flow segregation. Numerical examples are presented for direct and inverse problem.

Keywords: Granular media, segregation, conservation law, discontinuous flux, front tracking method, parameter identification

References

[1] R. Bürger. Introducción a Leyes de Conservación. *Centro de Investigación en Ingeniería Matemática, Universidad de Concepción*, 2021.

[2] L. May, M. Shearer, and K. Daniels. Scalar conservation laws with nonconstant coefficients with application to particle size segregation in granular flow. *J. Nonlinear Sci*, 20:689-707, 2010.
2008.

[3] H. Risebro. An introduction to the theory of scalar conservation laws with spatially discontinuous flux functions. *Applied Wave Mathematics: Selected Topics in Solids, Fluids, and Mathematical Methods*, 395-464, 2009.

CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Concepción, Chile
Email address: rburger@ing-mat.udec.cl

CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Concepción, Chile
Email address: yesmartinez@udec.cl

CI^{2}MA, Universidad de Concepción, and Departamento de Matemática, Universidad del Bío-Bío, Concepción, Chile
Email address: lvillada@ubiobio.cl