A MIXED FINITE ELEMENT METHOD FOR A REVERSE OSMOSIS MODEL

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Abstract. We develop and analyze a numerical method to approximate the solution to a partial differential equation arising from a phenomenological model of water desalination through reverse osmosis within a channel module. The problem involves a coupled nonlinear system, which considers the steady state of mass transport phenomena through a convection-diffusion equation and linear momentum balance via the Navier-Stokes equation. To address this problem, we introduce a mixed variational formulation for both phenomena, utilizing suitable Lebesgue spaces to define nonlinear terms, and introducing a Lagrange multiplier that couples both phenomena at the boundary. We establish existence and uniqueness of the solution under smallness assumptions on the physical parameters. We consider conforming subspaces, show well-posedness of the discrete formulation, and the respective *a priori* error estimates. Finally, we compare our results against benchmarks available in the literature to assess the proper functioning of the numerical method.

Keywords: desalinitaion process, mixed finite element method, *a priori* error analysis, reverse osmosis.

Mathematics Subject Classifications (2010): 65N30, 65N12, 65N15

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