TOTAL PRESSURE-BASED FREQUENCY-DOMAIN FORMULATION AND CONVERGENCE ANALYSIS OF BIOT'S POROELASTICITY EQUATIONS WITH A NEW FINITE ELEMENT STABILIZATION

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ABSTRACT. In time-harmonic imaging elastography, we delve into complex-variable-based poroelastic problems, wherein coefficients such as permeability may exhibit sudden spatial variations [?]. We explore their theoretical foundations, assess their practical implications, and consider their prospective applications as inverse problems. Our approach begins with the application of the well-established Fredholm Alternative to prove the solvability of the continuous problem. To overcome the challenges posed by the discrete inf-sup condition, we introduce an innovative and robust stabilized numerical system, optimized for equal polynomial order, which enhances computational efficiency. Additionally, we conduct a numerical evaluation to assess the stability of solutions relative to the permeability constant and offer an a priori analysis of these solutions. To elucidate our insights, we augment our theoretical discourse with numerical examples. These illustrations furnish compelling evidence of the practical applicability and efficacy of the proposed numerical framework. By bridging the theoretical complexities of complex-variable-based poroelastic problems with feasible solutions, our research contributes valuable insights for applications in Imaging Elastography and related disciplines.

 ${\bf Keywords:} \ {\rm Biot, \ poroelasticity, \ magnetic \ resonance \ elastography, \ stabilized \ finite \ element.}$

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