A NONCONFORMING FINITE ELEMENT METHOD FOR A NONISOTHERMAL FLUID-MEMBRANE INTERACTION

JESSIKA CAMAÑO, RICARDO OYARZÚA, MIGUEL SERÓN, AND MANUEL SOLANO

Abstract. We propose and analyze an $H(div)$-conforming and mass conservative finite element method for a nonisothermal fluid-membrane interaction problem. The problem consists of a Navier-Stokes/heat system, commonly known as the Boussinesq system, in the free-fluid region, and a Darcy-heat coupled system in the membrane. These systems are coupled through buoyancy terms and a set of transmission conditions on the fluid-membrane interface, including mass conservation, balance of normal forces, the Beavers-Joseph-Saffman law, and continuity of heat flux and fluid temperature. We consider the well-known velocity-pressure-temperature variational scheme for the Boussinesq system in the free-fluid region whereas in the membrane region we consider a dual-mixed formulation for the Darcy system coupled with a primal equation for the temperature model. In this way, the unknowns of the resulting formulation are given by the velocity, the pressure, and the temperature in both domains. For the associated Galerkin scheme, we combine an $\mathbf{H}(\mathrm{div})$-conforming scheme for the fluid variables and a conforming Galerkin discretization for the heat equation. Therefore, the resulting numerical scheme yields exactly divergence–free velocities. We prove well-posedness by means of a fixed–point strategy and derive the corresponding convergence. Finally, we present some numerical examples to illustrate the performance of the method.

Keywords: nonisothermal fluid-membrane, Navier–Stokes equation, Darcy equation, heat equation, discontinuous Galerkin methods.

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Universidad Católica de la Santísima Concepción
Email address: jecamano@ucsc.cl

Universidad del Bío-Bío
Email address: royarzua@ubiobio.cl

Universidad del Bío-Bío
Email address: mseron@ubiobio.cl

Universidad de Concepción
Email address: msolano@ing-mat.udec.cl