LOCAL MINIMUM-RESIDUAL A POSTERIORI ERROR ESTIMATES FOR A CLASS OF MIXED FINITE ELEMENT DISCRETIZATIONS

LILIANA CAMARGO, IGNACIO MUGA, SERGIO ROJAS, AND PATRICK VEGA

Abstract. We introduce reliable and efficient a posteriori error estimates for a class of mixed formulations to solve partial differential equations involving a diffusion term. It combines a superconvergent postprocessing technique for the primal variable with an adaptive finite element method via residual minimization. Such a residual minimization procedure is performed on a local postprocessing scheme, commonly used in mixed finite element methods. Given the local nature of such an approach, the underlying saddle point problems associated with residual minimizations can be solved with minimal computational effort. We propose and study a posteriori error estimators based on an improvement of the built-in residual representative associated with residual minimization schemes, which adds, on the one hand, a residual term quantifying the mismatch between discrete fluxes and, on the other hand, the interelement jumps of the post-processed solution. We present several numerical experiments in two dimensions, including a standard mixed formulation for advection-diffusion problems using Brezzi–Douglas–Marini elements and a mixed Hybridizable Discontinuous Galerkin (HDG) formulation for the Helmholtz equation. The experiments perfectly fit our key theoretical findings and suggest that our estimates are sharp.

Keywords: residual minimization, postprocessing, superconvergence, a posteriori error analysis, adaptive mesh refinement.

Mathematics Subject Classifications (2010): 65N12, 65N15, 65N22, 65N30, 65N50.

References

[1] L. Camargo, S. Rojas, and P. Vega. Minimum-residual a posteriori error estimates for a hybridizable discontinuous Galerkin discretization of the Helmholtz equation *arXiv preprint*, arXiv:2304.00418, 2023.

[2] I. Muga, S. Rojas, and P. Vega. An adaptive superconvergent mixed finite element method based on local residual minimization *SIAM Journal on Numerical Analysis*, 61(5):2084-2105, 2023.

Instituto de Matemáticas, Pontificia Universidad Católica de Valparaíso, Chile.
Email address: liliana.camargo@pucv.cl

Instituto de Matemáticas, Pontificia Universidad Católica de Valparaíso, Chile.
Email address: ignacio.muga@pucv.cl

Instituto de Matemáticas, Pontificia Universidad Católica de Valparaíso, Chile.
Email address: sergio.rojas.h@pucv.cl

Departamento de Matemática y Ciencia de la Computación, Universidad de Santiago de Chile, Chile.
Email address: patrick.vega@usach.cl