ROBUST FINITE ELEMENT METHODS AND SOLVERS FOR THE BIOT-BRINKMAN EQUATIONS IN VORTICITY FORM

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Abstract. This presentation discusses a novel approach and a finite element technique tailored for the stable coupling of viscous fluid flow within deformable porous materials using divergence-conforming filtration fluxes. The suggested method relies on utilizing spaces weighted by parameters, enabling a more precise and resilient examination of both continuous and discrete problems. Additionally, we perform an analysis of the method's solvability and establish optimal error estimates within suitable norms. These error estimates are demonstrated to remain reliable even when dealing with high Lamé parameters and low permeability and storativity coefficients. To showcase the effectiveness of this approach, we offer several illustrative numerical instances, including verification of convergence, simulation of poroelastic channel flow, and an assessment of the resilience of block-diagonal preconditioners concerning model parameters.

Keywords: Biot-Brinkman coupled problem; deformable porous media; vorticity-based formulation; mixed finite element methods.

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