COUPLED FINITE VOLUME AND MIXED FINITE ELEMENT METHODS FOR THE VISCOUS MODEL OF SEDIMENTATION

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Abstract. A numerical scheme combining finite volume and mixed finite element methods for the so-called viscous model of sedimentation is presented. A particular feature of the governing equations is given by the fact that the velocity field is non-divergence free. We introduce extra variables such as the pseudostress tensor relating the velocity gradient with the pressure, thus leading to a mixed variational formulation consisting of two systems of equations coupled through their source terms. A result of existence and uniqueness of solutions is shown by means of a fixed-point strategy and the help of the Babuška-Brezzi theory and Banach theorem. Additionally, we employ suitable finite dimensional subspaces to approximate both systems of equations via associated mixed finite element methods. The well-posedness of the resulting coupled scheme is also treated via a fixed-point approach, and hence the discrete version of the existence and uniqueness result is derived analogously to the continuous case. The above is then combined with a finite volume method for the transport equation. Finally, several numerical results illustrating the performance of the proposed model and the full numerical scheme, and confirming the theoretical rates of convergence, are presented.

Keywords: solid phase velocity, multidimensional sedimentation, mixed finite elements, fixed-point problem

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References

[1] J. Careaga, and G.N. Gatica. Coupled mixed finite element and finite volume methods for a solid velocity-based model of multidimensional settling. ESAIM: *Mathematical Modelling and Numerical Analysis*, 57(4):2529-2556, 2023.

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