

FROM DPG TO MIXED FEM

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ABSTRACT. Since its start more than a decade ago [2, 3], the discontinuous Petrov–Galerkin (DPG) method with optimal test functions has itself established as a well-understood and useful framework. As the name suggests, special test functions are used in a DG-setting; they aim at discrete stability. Essential for progress has been the systematic use of ultra-weak variational formulations. They transfer all appearing derivatives onto the test side, and generate trace terms by doing so, giving rise to independent trace variables [1].

Considering challenging problems, e.g., from the mechanics of thin structures, it becomes clear that corresponding trace spaces and operations are critical to design robust well-posed DPG schemes, see [5, 6]. On the other hand, trace operators are directly related with interface conditions and thus, characterize the conformity of corresponding fields. This is a relevant subject for the numerical analysis of PDE in general. In this talk we present some new results for mixed finite element methods that originate from DPG trace and discretization techniques, for plate bending [4] (collaboration with T. Führer) and linear elasticity (collaboration with C. Carstensen).

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