BANACH SPACE-BASED ANALYSIS OF A FULLY MIXED FORMULATION FOR THE NAVIER–STOKES/DARCY EQUATIONS

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Abstract. We propose and analyze a fully-mixed formulation to couple fluid flow with porous media flow, governed by the Navier–Stokes and Darcy equations, respectively. The transmission conditions consist of mass conservation, balance of normal forces, and the Beavers-Joseph-Saffman law. Our approach involves introducing a modified pseudostress tensor dependent on pressure, the diffusive and convective terms of the fluid Navier–Stokes equations, while employing the standard dual-mixed formulation for the Darcy model. The method results in a mixed variational formulation based on Banach spaces and a twofold saddle point structure. The key unknowns for the Navier-Stokes fluid are the pseudostress tensor, vorticity, and velocity, whereas the porous medium has velocity and pressure as its corresponding unknowns. We establish the well-posedness of both continuous and discrete formulations using a fixed-point strategy and the Banach–Nečas–Babuška and Banach's fixed point theorems. These results apply to arbitrary finite element subspaces under suitable stability assumptions. Additionally, we identify specific finite element subspaces that satisfy the necessary conditions and provide convergence analysis, demonstrating the method's optimal rate of convergence. Finally, we present several numerical results to illustrate the performance of the proposed method.

Keywords: Navier–Stokes, Darcy, momentum conservativity, mixed finite element method, Banach spaces, Arnold–Falk–Winther elements, Raviart–Thomas elements

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