

# A MIXED FEM FOR THE COUPLED BRINKMAN–FORCHHEIMER/DARCY PROBLEM

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ABSTRACT. This work develops the *a priori* analysis of a mixed finite element method for the filtration of an incompressible fluid through a non-deformable saturated porous medium with heterogeneous permeability. Flows are governed by the Brinkman–Forchheimer and Darcy equations in the more and less permeable regions, respectively, and the corresponding transmission conditions are given by mass conservation and continuity of momentum. We consider the standard mixed formulation in the Brinkman–Forchheimer domain and the dual-mixed one in the Darcy region, and we impose the continuity of the normal velocities by introducing suitable Lagrange multiplier. The finite element discretization involves Bernardi–Raugel and Raviart–Thomas elements for the velocities, piecewise constants for the pressures, and continuous piecewise linear elements for the Lagrange multiplier. Stability, convergence, and *a priori* error estimates for the associated Galerkin scheme are obtained. Numerical tests illustrate the theoretical results.

**Keywords:** Brinkman–Forchheimer problem, Darcy problem, pressure-velocity formulation, mixed finite element methods, *a priori* error analysis

**Mathematics Subject Classifications (2020):** 65N30, 65N12, 65N15, 74F10, 76D05, 76S05.

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