A POSTERIORI ERROR ESTIMATES FOR IPDG DISCRETIZATIONS OF HELMHOLTZ PROBLEMS WITH MINIMAL REGULARITY

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Abstract. Helmholtz problems model the propagation of time-harmonic waves, and naturally appear in a large array of applications. The design of suitable numerical approximations is therefore paramount, and due to their ability to handle complex propagation media, finite element and discontinuous Galerkin approximations have become very popular to numerically solve Helmholtz problems. In this context, *a posteriori* error estimation appears as an important tool both to accurately control the discretization error and to improve the efficiency by adaptively refining the mesh and/or the polynomial degree. For conforming finite element discretizations, equilibrated and residual-based *a posteriori* estimators have been respectively considered in [2] and [4], with an analysis that holds for very general domains and material coefficients, allowing for minimal regularity of the solution. For discontinuous Galerkin schemes with interior penalization (IPDG) however, the analysis presented in [3] and [5] only holds under a restrictive assumption on the regularity of the solution which excludes important problem configurations. In this talk, I will discuss sharp *a posteriori* error estimates for IPDG discretizations that hold under minimal regularity assumptions. These results have been recently developed in [1], and apply to both equilibrated and residual-based estimators. At the core of the proof lies a duality argument of Aubin–Nitsche type that allows for minimal solution regularity. Beyond its application in *a posteriori* error estimation for Helmholtz problems, this technique should be useful for analyzing other aspects of the IPDG method.

Keywords: A posteriori error estimates, Aubin–Nitsche trick, discontinuous Galerkin, Helmholtz problems, interior penalty, minimal regularity.

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