

# AN ENERGY-DISSIPATIVE DIFFUSIVE-INTERFACE HYBRIDIZABLE DISCONTINUOUS GALERKIN METHOD FOR TWO-PHASE FLOWS

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**ABSTRACT.** We propose a hybridizable discontinuous Galerkin (HDG) method for a diffusive-interface model of incompressible immiscible two-phase flows with surface tension and non-matching densities. Our method ensures energy dissipation and global mass conservation. Key to this is the divergence-free discretization of the model's entropy functional. This is achieved by employing an HDG method previously used for single-phase Navier-Stokes flows. Numerical examples validate our theoretical results.

**Keywords:** two-phase flow, energy dissipation, hybridizable discontinuous Galerkin

**Mathematics Subject Classifications (2010):** 65N30, 76T10

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