PARTICLE DYNAMICS WITH SHOCKS ON VORONOÏ MESHES

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ABSTRACT. As discussed in [1] for astrophysical applications, there always has been interesting questions at the intersection of standard discretization of PDEs in view of applications and the use of meshes with various given structures. See also the seminal work [4]. In this direction, we present a new first order numerical method [2] on lagrangian and moving Voronoï meshes for the numerical simulation of compressible flows with shocks and internal interfaces between different gas. The scheme can be seen either as a particle method (the mass of particles is constant and there is no connectivity between particles) or as specific finite volume solver.

The method is based on the closed form formula of the partial derivative of the volume of Voronoï cells with respect to the generators. The mathematical proof of the formula with partition functions comes from applied statistical physics [3], and it seems original with respect to the mathematical literature. A corollary is that the volume of Voronoï cells is generically of class C^1 with respect to the generators. The final scheme is conservative in local mass, total momentum and total energy, and it is endowed with an entropy inequality which insures the correctness of shocks calculations. Numerical illustrations in dimension $d = 2$ will be shown for basic problems on coarse meshes. The implementation developed to obtain the numerical illustrations uses a freely available library for the generation of the Voronoï cells at all time steps.

Other open problems will be discussed, in particular wether the closed form formula can be used for the discretization of other PDEs.

Keywords:

Mathematics Subject Classifications (2010):

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