A STRUCTURE-PRESERVING SCHEME FOR A HYPERBOLIC APPROXIMATION OF THE NAVIER-STOKES-KORTEWEG EQUATIONS

FIRAS DHAOUADI AND MICHAEL DUMBSER

ABSTRACT. We present an explicit second-order accurate structure-preserving finite volume scheme for the hyperbolic approximation of the Navier-Stokes-Korteweg equations. The model combines the unified Godunov-Peshkov-Romenski model of continuum mechanics with a recently proposed hyperbolic reformulation of the Euler-Korteweg system. The considered PDE system includes an evolution equation for a gradient field that is by construction endowed with a curl-free constraint. The numerical scheme presented here relies on the use of vertex-based staggered grids and is proven to preserve the curl constraint *exactly* at the discrete level, up to machine precision. We show evidence of this property via a set of numerical tests, including non-condensing bubbles as well as non-stationary Ostwald ripening test cases with several bubbles. We present quantitative and qualitative comparisons of the numerical solution, both, when the new structure-preserving discretization is applied and when it is not. In particular we show that some numerical solutions blow up in finite time when the curl-free constraint is not respected.

Keywords: Navier-Stokes-Korteweg system, hyperbolic systems, compressible multi-phase flows, structure-preserving curl-free schemes, staggered finite volume method, augmented Lagrangian approach,

Mathematics Subject Classifications (2010): 65M08; 35L40; 76M12; 76T10

REFERENCES

[1] Dhaouadi F, Dumbser M. 2023 A structure-preserving finite volume scheme for a hyperbolic reformulation of the Navier–Stokes–Korteweg equations. *Mathematics* **11**, 876.

[2] Dhaouadi F, Dumbser M. 2022 A first order hyperbolic reformulation of the Navier-Stokes-Korteweg system based on the GPR model and an augmented Lagrangian approach. *Journal of Computational Physics* **470**, 111544.

LABORATORY OF APPLIED MATHEMATICS, DICAM, UNIVERSITY OF TRENTO, VIA MESIANO 77, 38123 TRENTO, ITALY

Email address: firas.dhaouadi@unitn.it

LABORATORY OF APPLIED MATHEMATICS, DICAM, UNIVERSITY OF TRENTO, VIA MESIANO 77, 38123 TRENTO, ITALY

Email address: michael.dumbser@unitn.it