A MATHEMATICAL-ECOLOGICAL MODEL OF BIOFILM GROWTH IN SLOW SAND FILTERS

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ABSTRACT. Slow Sand Filtration (SSF) is a type of water purification system that consists of a sand bed that acts both as a physical filter and as a settling zone for biological agents to attach and develop a biofilm layer capable of acting as a biological filter. This biofilm layer, usually called *Schmutzdecke*, constitutes the heart of the system and develops in the top 2–5 cm of the sand, growing up to a couple of centimeters above it [4, 3], where it acts as the major pathogen removal mechanism in the filter.

Although SSF has been used for more than two centuries, with early uses reported in the UK in 1804 [2], a comprehensive PDE model for the entire filter is not yet readily available, with most models using either only ODEs or simple diffusion equations. Since it is almost impossible to measure concentrations inside the filter, as it would disrupt the ecosystem formed inside [1], a mathematical model must be developed from physical principles and be flexible enough to account for different ecological models and the change of bacterial population in the filter.

To that end, in this work we have developed a comprehensive model that considers physical mechanisms (e.g. attachment, diffusion, and adsorption) and biological ones (e.g. grazing, photosynthetic growth and death), taking also into account the viscoelastic properties of the biofilm, the movement and growth in the supernatant water of the filter is modeled by a Cahn-Hilliard type PDE. We have implemented a numerical scheme based on a finite differences discretization in space and a semi-implicit method in time, resulting in a splitting scheme between the Cahn-Hilliard type equation in the supernatant water and a system of balance laws throughout the filter. A positivity bound on the time step-size for the balance law system allows us to define an adaptive time step scheme to speed up computations. Simulation results are consistent with the qualitative description of the behaviour of a filter and some scenario simulations under different weather conditions are shown.

Keywords: biofilm, drinking water treatment, partial differential equations, slow sand filtration

Mathematics Subject Classifications (2010):

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