AN UNFITTED METHOD (ϕ-FEM) COMBINED WITH DEEP LEARNING: VARIABLE GEOMETRIES AND CORRECTION

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Abstract. ϕ-FEM is a recently proposed finite element method for the efficient numerical solution of partial differential equations posed in domains of complex shapes, using simple structured meshes, not necessarily fitted to the domain, and achieving the optimal accuracy [1, 2]. The method relies on the use of a level-set function ϕ, defining the domain and its boundary. In this talk, we shall present a way to combine ϕ-FEM with Fourier Neural Operators (FNO)[3]. FNO uses Fast Fourier Transform, so that the solution should be represented on a Cartesian grid. In this context, ϕ-FEM turns out to be a promising alternative for training a neural network to provide predictions under the varying applied forces and under the varying geometries, conveniently represented by level sets. The efficiency of this combination will be illustrated with some numerical results on the Poisson equation with Dirichlet boundary conditions and on varying shapes.
In a second part, the predictions of the neural network will be fed back into a finite element solver to apply a correction to improve the accuracy of the solution. We will present some numerical results for different correction methods: correction by adding or by multiplying on an elevated problem.

Keywords:Fourier Neural operator, Unfitted finite element method, level set, varying geometry, correction.

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