A MULTISCALE FIVE-FIELD COMPOSITE MIXED FINITE ELEMENTS FOR BIOT PROBLEMS BASED ON GENERAL POLYGONAL PARTITIONS

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Abstract. Biot's consolidation model describes the deformation of an elastic porous medium and the fluid flow inside when the porous medium is saturated by the fluid. The governing equations are Darcy's law for the fluid motion, whereas the deformation of porous media is governed by linear elasticity. We consider a five field mixed method for this problem computing stress, displacement, and rotation elasticity variables, and flux and pressure for the fluid flow. Rotation, displacement and pressure play the role of Lagrange multipliers associated with stress symmetry and divergence constraints [1, 2]. The methods are designed to cope with complex realistic problems requiring refined computations to capture small structures in the solutions, which usually imply elevated computational costs by standard discretizations. The approach is a combination of global-local principles characterizing the exact solution, which were previously explored for Darcy's flows and linear elasticity models in [3] and [4], respectively. The variables are searched in composite finite element spaces based on polygonal subdomains, formed by the conglomeration of local shape-regular micro partitions. The multiscale characteristic is expressed in terms of refined discretizations inside the subdomains versus coarser normal trace components of tensor (traction) and flux over subdomain boundaries, with respect to mesh size, polynomial degree, or both. General error estimates and stability are derived and a static condensation procedure is designed for computational implementation, with drastic reduction in the number of degrees-of-freedom. Numerical examples are simulated to attest convergence and robustness of the method.

Keywords: Mixed finite elements, Poroelasticity, Multiscale

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