# CONTINUOUSLY BOUNDS-PRESERVING LIMITING METHODS FOR HIGH-ORDER DISCONTINUOUS GALERKIN SCHEMES

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Abstract. The efficient and accurate computation of complex transport phenomena remains a driving force in the development of high-fidelity numerical discretization techniques. Within this broad field of research, the use of finite element methods such as discontinuous Galerkin (DG) schemes has grown in prevalence due to their high-order accuracy, geometric flexibility, and computational efficiency. However, their accuracy and robustness can become severely degraded when the physical systems in question must abide by strict constraints, which is frequently the case in multi-physics applications. Standard DG implementations do not generally guarantee that these constraints are satisfied by the discrete solution, which may result in inaccurate and physically inconsistent predictions or, more typically, the failure of the scheme altogether.

A typical remedy for this problem is to enforce these constraints via some form of *a posteriori* limiting on the discrete solution, combining the solution of the high-order scheme with a more robust, low-order scheme [1]. One critical drawback of these limiting approaches is that they are typically performed *discretely*, whereas the DG solution is represented piecewise continuously, such that the schemes are only provably bounds-preserving at the discrete spatial locations where the limiting was performed. For many applications, this is not sufficient – it is common in FEM to have to evaluate the solution at some other points which, in some cases, may not even be known *a priori* (e.g., in the remap stage of an arbitrary Lagrangian-Eulerian solver [2]).

In this talk, I will present a limiting technique for convex constraints in DG schemes which ensures that the solution is *continuously bounds-preserving* (i.e., across the entire solution polynomial) for any arbitrary choice of basis and approximation order. The proposed approach relies on utilizing the ability of DG schemes to preserve convex invariants on the element-wise mean [3] along with modified formulations of the constraint functionals which reduce the limiting operation to a single spatial minimization problem for each element in the mesh. As a result, the approach can be efficiently and straightforwardly implemented in general DG codebases and can be applied to a wide variety of governing equations and user-specified constraint functionals. The results of the proposed approach will be shown for high-order, unstructured DG schemes applied to nonlinear scalar transport problems and the compressible Euler equations.

**Keywords**: discontinuous Galerkin schemes, bounds preserving, limiting, structure preserving, hyperbolic conservation laws

**Mathematics Subject Classifications (2010)**: 65M60, 65M12, 35L65

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