FINITE ELEMENT SOLUTION OF THE REYNOLDS-ORR ENERGY EIGENVALUE PROBLEM

FEDERICO FUENTES AND WILMAR IMBACHÍ

Abstract. We introduce a convergence analysis of the finite element method applied to the Reynolds-Orr eigenvalue problem in wall-bounded shear-driven incompressible flows with arbitrary cross-section. The Reynolds-Orr eigenproblem can be written as a mixed formulation similar to Stokes flow, but including an extra term involving the strain rate tensor of the underlying laminar flow. The analysis of the resulting discrete eigenproblem must be adapted to the standard spectral approximation framework, since one of the bilinear forms which is coercive in the Stokes equations is no longer coercive. We demonstrate that the proposed approach delivers accurate estimates of errors associated with both eigenvalues and eigenfunctions. We carry out various numerical tests to showcase how well the method performs and to confirm the accuracy of our theoretical results.

Keywords: Reynolds-Orr eigenvalue problem; Stokes eigenvalue problem; Spectral problems; Error estimates.

Mathematics Subject Classifications (2010): 65N12, 65N15,65N25, 65N30; 76D07, 35Q35.

References

[1] Reynolds, O. (1895). IV. On the dynamical theory of incompressible viscous fluids and the determination of the criterion. *Philosophical Transactions of the Royal Society of London.*(a.), (186), 123-164.

[2] Orr, W. M. F. (1907, January). The stability or instability of the steady motions of a perfect liquid and of a viscous liquid. Part II: A viscous liquid. In *Proceedings of the Royal Irish Academy. Section A: Mathematical and Physical Sciences* (Vol. 27, pp. 69-138). Royal Irish Academy.

[3] Drazin, P. G., & Reid, W. H. (2004). *Hydrodynamic stability.* Cambridge University Press.

[4] Doering, C. R., & Gibbon, J. D. (1995). *Applied analysis of the Navier-Stokes equations* (No. 12). Cambridge University Press.

[5] Serrin, J. (1959). On the stability of viscous fluid motions. *Archive for Rational Mechanics and Analysis*, 3, 1-13.

[6] Fuentes, F., Goluskin, D., & Chernyshenko, S. (2022). Global stability of fluid flows despite transient growth of energy. *Physical Review Letters*, 128(20), 204502.

[7] Boffi, D. (2010). Finite element approximation of eigenvalue problems. *Acta Numerica*, 19, 1-120.

[8] Boffi, D., Brezzi, F., & Gastaldi, L. (1997). On the convergence of eigenvalues for mixed formulations. *Annali della Scuola Normale Superiore di Pisa-Classe di Scienze*, 25(1-2), 131-154.

[9] Joseph, D. D., & Carmi, S. (1969). Stability of Poiseuille flow in pipes, annuli, and channels. *Quarterly of Applied Mathematics*, 26(4), 575-599.

[10] Goulart, P. J., & Chernyshenko, S. (2012). Global stability analysis of fluid flows using sum-of-squares. *Physica D: Nonlinear Phenomena*, 241(6), 692-704.

[11] Joseph, D. D. (2013). *Stability of fluid motions I* (Vol. 27). Springer Science & Business Media.

Pontificia Universidad Católica de Chile, Institute for Mathematical and Computational Engineering (IMC), Santiago, Chile.
Email address: federico.fuentes@uc.cl

Pontificia Universidad Católica de Chile, Faculty of Mathematics, Santiago, Chile.
Email address: waimbachi@mat.uc.cl